'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2024

DSE-P4-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains DSE4A and DSE4B.
The candidates are required to answer any *one* from *two* papers.
Candidates should mention it clearly on the Answer Book.

DSE4A

BOOLEAN ALGEBRA AND AUTOMATA THEORY

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

   (a) Give an example of a bijective mapping between two ordered sets which is not an order isomorphism.

   (b) Draw an Automata that performs Binary Addition.

   (c) Is the Cartesian product of two lattices always a lattice? Prove your claim.

   (d) Construct a logic circuit which realize the logical operator "conditional, $\longrightarrow$".

   (e) Reduce the Boolean term $((x_1 + x_2) \cdot (x_1' + x_3))'$ to DNF.

   (f) Let $\sum = \{0, 1\}$ and $T = \{\omega \in \sum^* : \omega$ contains even number of $1's\}$. Show that T is an accepted language.

GROUP-B

Answer any *four* questions 6×4 = 24

2. Let L be a lattice. Then show that $a \wedge b = a$ if and only if $a \vee b = b$.

3. Minimize the following Boolean function.

$$F = ABC'D' + ABC'D + AB'C'D' + AB'CD + ABCD' + AB'CD'$$

4. (a) Define alphabet, string and language. 3+3

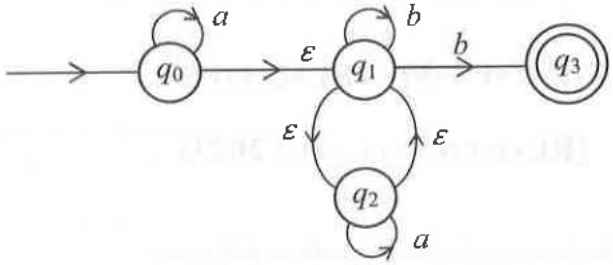
   (b) Write difference between deterministic finite automata and non-deterministic finite automata.

5. Suppose a 4-variable Boolean term is given as follows:

$$\phi = \sum m\,(0, 1, 2, 5, 7, 8, 9, 10, 13, 15)$$

Minimize ϕ using Karnaugh map.

6. (a) Find epsilon closures of all the states of the given ε-NFA.



(b) Distinguish between NFA and ε-NFA.

7. Simplify the following Boolean functions and then draw circuit in each case: 3+3

(i) $(x \cdot y + z') \cdot (y' + z \cdot x') + x' \cdot y' \cdot z'$

(ii) $[x \cdot y \cdot (x' \cdot y + x \cdot y')]'$.

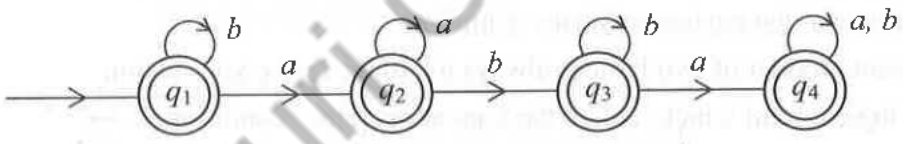
GROUP-C

Answer any *two* questions 12×2 = 24

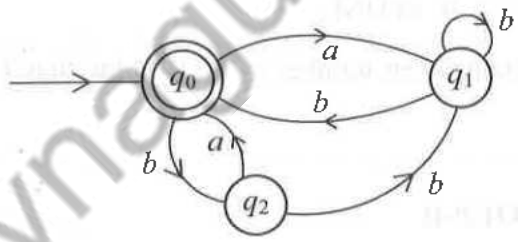
8. (a) Prove that a language L is accepted by some DFA iff L is accepted by some NFA. 6+3+3

(b) Find regular expression for the following DFAs:

(i)



(ii)



9. (a) Let L be a lattice. Then prove that the following statements are equivalent: 6+6

(i) For all $a, b, c \in L$; $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

(ii) For all $p, q, r \in L$; $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$.

(b) Design a Turing machine that accepts the language $L = \{O^{2^n} : n \geq 0\}$.

10.(a) A committee of three members decided that only proposal in the meeting will be approved only by the majority vote. Each member has a switch for casting vote by pressing the switch attached to their seats. Construct a circuit in such a way that it will allow current to pass when and only when a proposal is approved. 6+6

(b) Find the circuit that represents the following Boolean function. Also find a simpler circuit.

x	y	z	$f(x, y, z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

11.(a) Draw the transition diagram of FSA that accepts the given set of non-null strings over $\{a, b\}$. 4+4+4

(i) ends with *baa*

(ii) at least 2 *a*'s.

(b) Design a Turing machine that accepts the set of all strings over {0, 1} ending with 010.

(c) Show that the intersection of a context free language L and a regular language R is context free language.

DSE4B

MATHEMATICAL MODELLING

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) What is the difference between continuous and discrete models? Support your answer by suitable examples of each type. 3

(b) Suppose a population $p(t)$ at time t follows the growth rate equation $\frac{dp}{dt} = (a-b)p$ where a and b are respectively birth and death rates. Discuss the long term behaviour of the population considering $a >= < b$. 3

(c) Define a chemostat. Give an example of a bacterial growth model in a chemostat. 3

(d) Using Laplace transform find the value of $\int_0^\infty \sin\sqrt{t}\, dt$. 3

(e) In an archeological wooden specimen only 25% of original radio carbon 12 is present. When it was made? 3

(f) Find the output which yields maximum profit when the cost function is $0.3q^3 - 0.8q^2 + 12q + 5$, assuming the price of q is ₹ 30 per unit. 3

GROUP-B

Answer any *four* questions 6×4 = 24

2. Answer any ***two*** questions:

   (a) Find $\mathcal{L}^{-1}\left(\frac{2s^2}{(s^2+1)(s-1)^2}\right)$, if exists. 3

   (b) Find $\mathcal{L}(t^2 \sinh \omega t)$, if exists. 3

   (c) Find $\mathcal{L}(2te^{t^2}\cos(e^{t^2}))$, if exists. 3

3. (a) For a certain substance the rate of change of vapour pressure (P) w.r.to the temperature (T) is proportional to the vapour pressure and inversely proportional to the square of the temperature. Express this in a model. Show that $P(t) = ce^{-k/T(t)}$ where c and k are arbitrary constant. 3

   (b) Discuss the following market which is characterized by lagged supply response: 3

   $$D_t = 30 - 9P_t \;\; ; \;\; S_t = 3 + 12P_{t-1}$$

   the symbols have their usual meanings.

4. Suppose that the population $x(t)$ and $y(t)$ satisfy the model given by the system of equations: 6

   $$\frac{dx}{dt} = 14x - 2x^2 - xy$$

   $$\frac{dy}{dt} = 16y - 2y^2 - xy$$

   Determine all the critical points of the system and discuss the type and stability of these critical points. Which critical point represents the possibility of co-existence of the two populations?

5. The charge q of a condenser, capacity c, discharged in a circuit of resistance R, and self-inductance L satisfies the equation 6

   $$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

   Given that $q = q_0$ and $\frac{dq}{dt} = 0$ when $t = 0$ and that $CR^2 < 4L$. Use Laplace transform to find $q = q(t)$.

6. Write down the generating function for the Legendre polynomial. Show that 2+4

   $$(2n+1)x\,P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x),$$

   where $P_n(x)$ is the Legendre polynomial of degree n.

7. A and B play a game in which each has three coins ₹ 5, ₹ 10 and ₹ 20. Each select a coin without the knowledge of the other's choice. If the sum of the coins be an odd amount, then A wins B's coin; if the sum be even, then B wins A's coins. Find the best strategy for each player and the value of the game. 6

GROUP-C

Answer any *two* questions 12×2 = 24

8. (a) Solve in series the Bessel's equation near $x=0$; $x^2y''+xy'+(x^2-4)y=0$. 6

(b) Fit a parabolic curve of second degree to the given data: 5+1

Year	1973	1974	1975	1976	1977
Sales (in'000 ₹)	10	12	13	10	8

Also, estimate the value for 1986 and comment on it.

9. (a) Write the algorithm of Middle square method. What is the significance of this method in science? Use this method to generate 10 random numbers using $n_0=1009$. 2+1+3

(b) Assuming that the wind velocity u is in only one direction, the equation describing the dispersion of pollutants of concentration $c(x, t)$ is given by the equation: 6

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2} \quad , \quad 0<x<h \quad , \quad t>0$$

where D is the diffusion coefficient of the pollutants and $0<x<h$ is the region of interest. The initial and boundary conditions are given by $c(x, 0)=0$, $c(0, t)=c_1$, $c(h, t)=c_2$. Find $c(x, t)$.

10.(a) Solve the initial value problem using Laplace transformation: 6

$$\{tD^2+(1-2t)D-2\}y=0 \quad , \quad D\equiv\frac{d}{dt},$$

where $y(0)=1$ and $y'(0)=2$.

(b) Using Monte Carlo simulation calculate approximately the volume of an ellipsoid 6

$$\frac{x^2}{2}+\frac{y^2}{4}+\frac{z^2}{8}\le 16$$

that lies in the first octant, $x>0$, $y>0$, $z>0$.

11. Suppose there are two population group namely resource and consumer. Growth rate of the resource follows exponential law in absence of consumer and it decreases with respect to consumer's direct interaction, where as the consumers growth rate depends on how it consumes the resources which was decreases from resource's rate equation and finally consumer's death rate is directly proportional to self population size. Based on this, formulate a model in ordinary differential equation system and hence discuss the stability of its fixed point. 6+6

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