'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2024

DSE-P4-MATHEMATICS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains DSE4A and DSE4B.
The candidates are required to answer any *one* from *two* papers.
Candidates should mention it clearly on the Answer Book.

DSE4A

THEORY OF EQUATIONS

GROUP-A

1. Answer any ***four*** questions: $3\times4 = 12$

(a) Express the polynomial $8x^3+2x+2$ as a polynomial in $(2x-1)$. 3

(b) Apply Descartes' rule of signs to show that the equation $x^4+2x^2-7x-5=0$ has two real roots and two imaginary roots. 3

(c) Find the condition that the cubic $x^3-px^2+qx-r=0$ should have its roots in G.P. 3

(d) Find the special roots of the equation $x^6-1=0$. 3

(e) Find the equation whose roots are the cubes of the roots of the equation $x^3+3x^2+2=0$. 3

(f) If $\alpha,\ \beta,\ \gamma$ be the roots of the equation $x^3+px^2+qx-r=0$, then form the equation whose roots are $\beta\gamma+\dfrac{1}{\alpha},\ \gamma\alpha+\dfrac{1}{\beta},\ \alpha\beta+\dfrac{1}{\gamma}$. 3

GROUP-B

Answer any *four* questions $6\times4 = 24$

2. Prove that the roots of the equation $(x+2)(x-3)(x+4)+(x+1)(x+3)(x-5)=0$ are all real and distinct. Separate the intervals in which the roots lie. 6

3. If α be a special root of the equation $x^{12}-1=0$, prove that 6

$$(\alpha+\alpha^{11})(\alpha^5+\alpha^7)=-3$$

4. Solve the equation by Cardan's method $x^3-18x-35=0$. 6

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5. Show that the equation $x^3 - 3x + k = 0$ has three distinct real roots, if $-2 < k < 2$. 6

6. Solve the equation, given that it has multiple roots, $x^4 + 2x^3 + 2x^2 + 2x + 1 = 0$. 6

7. If α, β, γ be the roots of the cubic equation $x^3 - 21x + 35 = 0$, then show that $(\alpha^2 + 2\alpha - 14)$ is equal to either β or γ. 6

GROUP-C

Answer any *two* questions $12\times2 = 24$

8. (a) Find the number and position of real roots of the equation 6
$$x^4 - 6x^3 + 10x^2 - 6x + 1 = 0$$

(b) If a, b, c be the roots of the equation $x^3 + qx + r = 0$, then show that 6
$$a^5 + b^5 + c^5 + 5abc(bc + ca + ab) = 0$$

9. (a) Solve the equation by Ferrari's method: $x^4 - 6x^2 + 16x - 15 = 0$ 6

(b) If α be a root of the equation $x^3 - 3x - 1 = 0$, prove that the other roots are $2 - \alpha^2$, $\alpha^2 - \alpha - 2$. 6

10.(a) If α be an imaginary root of the equation $x^7 - 1 = 0$, find the equation whose roots are $\alpha + \alpha^6$, $\alpha^2 + \alpha^5$, $\alpha^3 + \alpha^4$. 6

(b) Solve the reciprocal equation $x^4 - 4x^3 + 3x^2 - 4x + 1 = 0$. 6

11.(a) The equation $3x^3 + 5x^2 + 5x + 3 = 0$ has three distinct roots of equal moduli. Solve it. 6

(b) Use Sturm's theorem to show that the equation $x^4 - 3x^3 - 2x^2 + 7x + 3 = 0$ has one root between (–2) and (–1), one root between (–1) and 0 and two roots between 2 and 3. 6

DSE4B

DIFFERENTIAL GEOMETRY

GROUP-A

1. Answer any *four* questions: $3\times4 = 12$

(a) Check whether the curve $\gamma(t) = \left(1 - t, \dfrac{1 + t^2}{t}, \dfrac{1 + t}{t}\right)$ is planer or not.

(b) Prove that the curve $x = 2\sin^2 t$, $y = 2\sin t \cos t$, $z = 2\cos t$ lies on a sphere.

(c) Find the reparametrization of $\gamma(t) = \left(t^2, \dfrac{t^3}{\sqrt{1 - t^2}}\right)$, $-1 < t < 1$.

(d) Find the radius of curvature for the curve $\gamma(t) = (2t, 3t^2, 2(t^3 + 1))$.

(e) Show that $x^2 + y^2 + z^4 = 1$ is a smooth surface.

(f) Find the evolute of the curve $x = a\cos t$, $y = a\sin t$, $z = a\cot bt$ where $a \neq 0$ and $b \neq \frac{\pi}{2}$ are constants.

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GROUP-B

Answer any *four* questions $6\times4=24$

2. Prove that the curve $u+v=\text{constant}$ are geodesic on a surface with the metric $(1+u^2)\,du^2-2uv\,dudv+(1+v^2)\,dv^2$.

3. Find the first fundamental form of $\sigma(\theta,\varphi)=(\operatorname{sec}h\theta\cos\varphi,\ \operatorname{sec}h\theta\sin\varphi,\ \tan\theta)$.

4. Find the Gaussian curvature for the surface
$$\sigma(u,v)=(-\cosh u\cos v,\ -\cosh u\sin v,\ \sin u)$$

5. Find the standard unit normal of
$$\sigma(\theta,\ \varphi)=((a+b\cos\theta)\cos\varphi,\ (a+b\cos\theta)\sin\varphi,\ b\sin\theta)$$
where a, b are constants.

6. Define developable surface. Find the conditions for a surface $z=f(x,y)$ to be a developable surface. 1+5

7. For the curve $\gamma(t)$, prove that $[\gamma'\ \gamma''\ \gamma''']=\dfrac{[\dot{r}\,\ddot{r}\,\dddot{r}]}{\dot{r}^6}$.

GROUP-C

Answer any *two* questions $12\times2=24$

8. (a) Define involute of a curve. Prove that the involute of $r(t)=(t,\cosh t)$ is $x=\cosh^{-1}(\frac{1}{y})-\sqrt{1-y^2}$. 6

   (b) Calculate the second fundamental form of the surface. 6
$$\sigma(u,v)=(u\cos v,\ u\sin v,\ v)$$

9. (a) Define asymptotic lines. Prove that parametric curve on a surface $\sigma(u,\ v)=(u\cos v,\ u\sin v,\ v)$ are asymptotic line. 2+6

   (b) Find the arc length of the curve $x=3\cosh 2t$, $y=3\sinh 2t$, $z=6t$ from $t=0$ to $t=\pi$. 4

10. (a) Prove that the geodesic curvature of a geodesic on a surface is zero and conversely. 5

    (b) Find the parametric representation of $x^3+y^3+3xy=0$. 3

    (c) Show that the radius of spherical curvature of a circular helix $x=a\cos\theta$, $y=a\sin\theta$, $z=a\theta\cot\alpha$ is equal to the radius of circular curvature. 4

11. State Frenet-Serret equation verify it for the curve 2+10
$$r(t)=\left(\sin^2\frac{t}{\sqrt{2}},\ \frac{1}{2}\sin t\sqrt{2},\ \frac{t}{\sqrt{2}}\right)$$

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