'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2024

DSE-P3-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains DSE-3A and DSE-3B. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.

DSE-3A

POINT SET TOPOLOGY

GROUP-A

1. Answer any ***four*** from the following questions: $3\times 4=12$

(a) Let $f:X\to Y$ be a continuous function and $\phi\neq A\subset X$. If $x\in\overline{A}$, does $f(x)$ necessarily belong to $\overline{f(A)}$? Justify your answer. 3

(b) Let $F(X)$ be the collection of all finite subsets of X. If X is infinite, then prove that $|F(X)|=|X|$, where $|\cdot|$ denotes the cardinality. 3

(c) If $X=\{a,\ b,\ c\}$. Let $\tau_1=\{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$; $\tau_2=\{\phi,\ X,\ \{a\},\ \{b,\ c\}\}$. Find the smallest topology containing τ_1 and τ_2 and the largest topology contained in τ_1 and τ_2. 3

(d) Let A be a connected subspace of X and $A\subset B\subset\overline{A}$, then show that B is also connected. 3

(e) Show that no two spaces from $(0,\ 1)$, $(0,\ 1]$ and $[0,1]$ are homeomorphic. 3

(f) Let $(X,\ T)$ be a topological space, where $X=\{a,\ b,\ c\}$, 3

$$\Gamma=\{\phi,\ X,\ \{a\},\ \{a,\ b\},\ \{b,\ c\}\}$$

Find the closure and interior of A, where $A=\{a,\ c\}$.

GROUP-B

Answer any *four* questions from the following $6\times 4=24$

2. Prove that $2^a=c$, where $|\mathbb{N}|=a$ and $|\mathbb{R}|=c$. 6

3. Let $f:(X, \Gamma_X) \rightarrow (Y, \Gamma_Y)$ be a mapping prove that the following are equivalent: (i) f is continuous; (ii) $f(\overline{A}) \subset \overline{f(A)}$ for all $A \subset X$; (iii) for any closed set c in Y, $f^{-1}(c)$ is closed in X. 6

4. Show that $\beta = \{(a, b) : a, b \in \theta\}$ is a countable basis of $\mathbb{R}$. 6

5. Show that a topological space X is compact if and only if every collection of closed sets in X satisfying the finite intersection property has a non-empty intersection. 6

6. Show that if U is open, connected subset of $\mathbb{R}^2$, then U is path connected. 6

7. Show that the product topology on $X \times Y$ is the weakest topology on $X \times Y$ determined by the projection maps π_1 and π_2. 6

GROUP-C

Answer any *two* questions from the following $12\times 2 = 24$

8. (a) Prove that if X is an order set in which every closed interval is compact in the order topology, then X has the least upper bound property. 6

   (b) Let X, Y be topological spaces and Y be compact Hausdorff. Let $f: X \rightarrow Y$ be a function. Show that f is continuous if and only if the graph of f

$$\Gamma_f = \{(x, f(x)) : x \in X\}$$

   is closed in $X \times Y$. 6

9. (a) Show that $\mathbb{R}^n$ and $\mathbb{R}^m$ can not be homeomorphic if $m \neq n$. 6

   (b) Is any infinite subset of $\mathbb{R}$ compact with respect to discrete topology on $\mathbb{R}$? Justify your answer with proper explanation. 6

10. (a) Let X be a compact Hausdorff space and let $\{A_n\}$ be a countable collection of closed sets in X. Show that if each set A_n has empty interior in X then the union $\bigcup_n A_n$ also has empty interior in X. 7

   (b) Let Y be a subspace of X and A be a subset of Y. Let $\overline{A}$ be the closure of A in X. Prove that the closure of A in Y is $\overline{A} \cap Y$. 5

11. (a) Let $X_1, X_2, \cdots, X_n$ be topological spaces. For each $k \in \{1, 2, \cdots, n\}$, define the k^{th} coordinate projection map π_k as follows 3

$$\pi_k : X_1 \times X_2 \cdots \times X_n \rightarrow X_k \quad \text{given by} \quad \pi_k(x_1, x_2, \cdots, x_n) = x_k$$

   Show that the projection maps are open maps.

   (b) If a is a cardinal, then prove that $a \leq a + 1$. 3

   (c) Show that every compact subspace of a Hausdorff space is closed. 6

DSE-3B

LINEAR PROGRAMMING

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Make a graphical representation of the set of constraints in the following L.P.P. 3

$$\text{Max} \quad Z = 2x_1 + x_2$$

$$\text{Subject to} \quad x_1 + 3x_2 \le 15$$

$$3x_1 - 4x_2 \le 12$$

$$x_1,\ x_2 \ge 0$$

and hence find the maximum value of the objective function by using moving hyperplane method.

(b) Prove that in E^2, the set $X = \{(x,\ y):\ y^2 \le x\}$ is a convex set. 3

(c) Prove that a hyperplane is a convex set. 3

(d) Prove that $x_1 = 3$, $x_2 = 0$, $x_3 = 0$ is a feasible solution to the following set of equations 3

$$4x_1 + x_2 - 3x_3 = 12$$

$$6x_1 + 3/2\,x_2 + x_3 = 18$$

Is the solution basic? If so, which are the basic variables?

(e) Prove that if the primal problem has feasible solution and dual has no feasible solution then the primal problem is said to have unbounded solution. 3

(f) Find an initial basic feasible solution of the following transportation problem by cost minima method. 3

	D_1	D_2	D_3	D_4	a_i
O_1	2	1	3	4	30
O_2	3	2	1	4	50
O_3	5	2	3	8	20
b_j	20	40	30	10	

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Prove that extreme points are finite in number. 6

3. Show that (2, 1, 3) is a feasible solution of the set of equations 6

$$4x_1 + 2x_2 - 3x_3 = 1$$

$$6x_1 + 4x_2 - 5x_3 = 1$$

Reduced it to a basic feasible solution of the system.

4. Solve the following LPP by Simplex method: 6

Max $Z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

$x_1,\ x_2,\ x_3 \ge 0$

5. Prove that in a balance Transportation problem having m origins and n destinations $(m,\ n \ge 2)$ the exact number of basic variable is $m+n-1$. 6

6. Find the optimal assignment for a problem with the following cost matrix 6

	M_1	M_2	M_3	M_4	M_5
J_1	8	4	2	6	1
J_2	0	9	5	5	4
J_3	3	8	9	2	6
J_4	4	3	1	0	3
J_5	9	5	8	9	5

7. In a rectangular game the pay-off matrix is given by 6

A \ B	B_1	B_2	B_3	B_4	B_5
A_1	10	5	5	20	4
A_2	11	15	10	17	25
A_3	7	12	8	9	8
A_4	5	13	9	10	5

Find the value of the game.

GROUP-C

Answer any *two* questions from the following $12 \times 2 = 24$

8. (a) Prove that the set of all convex combinations of a finite number of points is a convex set. 6

(b) Solve the following L.P.P. by usual Simplex Method without using any artificial variable. 6

Minimize $Z = x_1 + x_2$

Subject to $x_1 + 2x_2 \ge 12$

$5x_1 + 6x_2 \ge 48$

$x_1,\ x_2 \ge 0$

9. (a) Prove that dual of the dual is the primal itself. 6

(b) Obtain an optimal basic feasible solution to the following transportation problem. 6

	W_1	W_2	W_3	W_4	a_i
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
b_j	5	8	7	14	

10.(a) Solve by two-phase method: 7

$$\begin{aligned} \text{Max} \quad & Z = 2x_1 + x_2 - x_3 \\ \text{Subject to} \quad & 4x_1 + 6x_2 + 3x_3 \le 8 \\ & 3x_1 - 6x_2 - 4x_3 \le 1 \\ & 2x_1 + 3x_2 - 5x_3 \ge 4 \\ & x_1,\ x_2,\ x_3 \ge 0 \end{aligned}$$

(b) A vector is basic feasible solution of $Ax = b$ if and only if it is an extreme point of the set $\{x : Ax = b,\ x \ge 0\}$. 5

11.(a) Prove that every two-person zero sum game can be converted to a L.P.P. 6

(b) Solve the following game problem by reducing 2×2 problem using dominance property. 6

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 & B_4 \end{matrix} \\ \begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \end{array}$$

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