'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2024

DSE-P3-MATHEMATICS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains DSE-3A and DSE-3B. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.

DSE-3A

POINT SET TOPOLOGY

GROUP-A

1. Answer any ***four*** from the following questions: $3\times4=12$

(a) Let $f:X\to Y$ be a continuous function and $\phi\neq A\subset X$. If $x\in\overline{A}$, does $f(x)$ necessarily belong to $\overline{f(A)}$? Justify your answer. 3

(b) Let $F(X)$ be the collection of all finite subsets of X. If X is infinite, then prove that $|F(X)|=|X|$, where $|\cdot|$ denotes the cardinality. 3

(c) If $X=\{a,\ b,\ c\}$. Let $\tau_1=\{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$; $\tau_2=\{\phi,\ X,\ \{a\},\ \{b,\ c\}\}$. Find the smallest topology containing τ_1 and τ_2 and the largest topology contained in τ_1 and τ_2. 3

(d) Let A be a connected subspace of X and $A\subset B\subset\overline{A}$, then show that B is also connected. 3

(e) Show that no two spaces from (0, 1), (0, 1] and [0, 1] are homeomorphic. 3

(f) Let $(X,\ T)$ be a topological space, where $X=\{a,\ b,\ c\}$, 3

$$\Gamma=\{\phi,\ X,\ \{a\},\ \{a,\ b\},\ \{b,\ c\}\}$$

Find the closure and interior of A, where $A=\{a,\ c\}$.

GROUP-B

Answer any *four* questions from the following $6\times4=24$

2. Prove that $2^a=c$, where $|\mathbb{N}|=a$ and $|\mathbb{R}|=c$. 6

3. Let $f:(X, \Gamma_X)\to(Y, \Gamma_Y)$ be a mapping prove that the following are equivalent: (i) f is continuous; (ii) $f(\overline{A})\subset\overline{f(A)}$ for all $A\subset X$; (iii) for any closed set c in Y, $f^{-1}(c)$ is closed in X. 6

4. Show that $\beta=\{(a, b):\ a, b\in\theta\}$ is a countable basis of $\mathbb{R}$. 6

5. Show that a topological space X is compact if and only if every collection of closed sets in X satisfying the finite intersection property has a non-empty intersection. 6

6. Show that if U is open, connected subset of $\mathbb{R}^2$, then U is path connected. 6

7. Show that the product topology on $X\times Y$ is the weakest topology on $X\times Y$ determined by the projection maps π_1 and π_2. 6

GROUP-C

Answer any *two* questions from the following $12\times2=24$

8. (a) Prove that if X is an order set in which every closed interval is compact in the order topology, then X has the least upper bound property. 6

   (b) Let X, Y be topological spaces and Y be compact Hausdorff. Let $f:X\to Y$ be a function. Show that f is continuous if and only if the graph of f 6

   $$\Gamma_f=\{(x,\ f(x)):\ x\in X\}$$

   is closed in $X\times Y$.

9. (a) Show that $\mathbb{R}^n$ and $\mathbb{R}^m$ can not be homeomorphic if $m\neq n$. 6

   (b) Is any infinite subset of $\mathbb{R}$ compact with respect to discrete topology on $\mathbb{R}$? Justify your answer with proper explanation. 6

10. (a) Let X be a compact Hausdorff space and let $\{A_n\}$ be a countable collection of closed sets in X. Show that if each set A_n has empty interior in X then the union $\bigcup_n A_n$ also has empty interior in X. 7

   (b) Let Y be a subspace of X and A be a subset of Y. Let $\overline{A}$ be the closure of A in X. Prove that the closure of A in Y is $\overline{A}\cap Y$. 5

11. (a) Let $X_1,\ X_2,\cdots,\ X_n$ be topological spaces. For each $k\in\{1,\ 2,\cdots,\ n\}$, define the k^{th} coordinate projection map π_k as follows 3

   $$\pi_k:\ X_1\times X_2\cdots\times X_n\to X_k \quad \text{given by} \quad \pi_k(x_1,\ x_2,\cdots,\ x_n)=x_k$$

   Show that the projection maps are open maps.

   (b) If a is a cardinal, then prove that $a\leq a+1$. 3

   (c) Show that every compact subspace of a Hausdorff space is closed. 6

DSE-3B

BOOLEAN ALGEBRA AND AUTOMATA THEORY

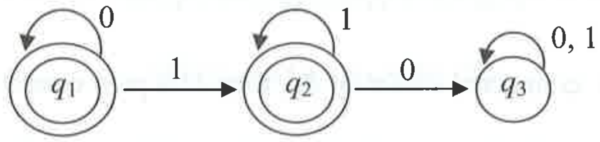
GROUP-A

1. Answer any ***four*** questions from the following: $3\times4 = 12$

(a) Design a DFA that accepts the language

$$L = \{x \in \{a,\ b\}^* :\ x \text{ ends with } aba\}.$$

(b) Describe in English the set accepted by the finite automata whose transition diagram.



(c) Write the dual of each statement:

(i) $(a \wedge b) \vee c = (b \vee c) \wedge (c \vee a)$

(ii) $(a \wedge b) \vee a = a \wedge (b \vee a)$

(d) Consider the strings $u = a^2ba^3b^2$ and $v = bab^2$. Find (i) uv (ii) v^2.

(e) Using Boolean algebra, show that

$$(x \cdot y)(x' \cdot z' + z) \cdot (x \cdot (z + y)') = 0$$

(f) For any two elements $x,\ y$ in a lattice, let

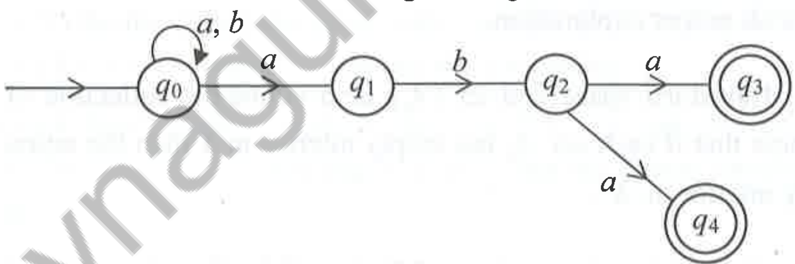
$$[x,\ y] = \{a \in L :\ x \le a \le y\}$$

Show that $[x,\ y]$ is a sublattice of L.

GROUP-B

2. Answer any ***four*** questions from the following: $6\times4 = 24$

(a) Let M be the NFA whose state diagram is given below: 6



Write down the transition table for this NFA. Also find $L(M)$.

(b) Define alphabet, string and language. Write difference between deterministic finite automata and non-deterministic finite automata. 6

(c) In a distributive lattice $(A, \le)$, if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a, then show that $x = y$. 6

(d) For $\Sigma = \{a, b, c\}$, design a turning machine that accepts $L = \{a^n b^n c^n \mid n \ge 1\}$. 6

(e) Draw a switching circuit for the Boolean expressions 3+3

(i) $x(yz + y'z') + x'(yz + y'z + z'y)$

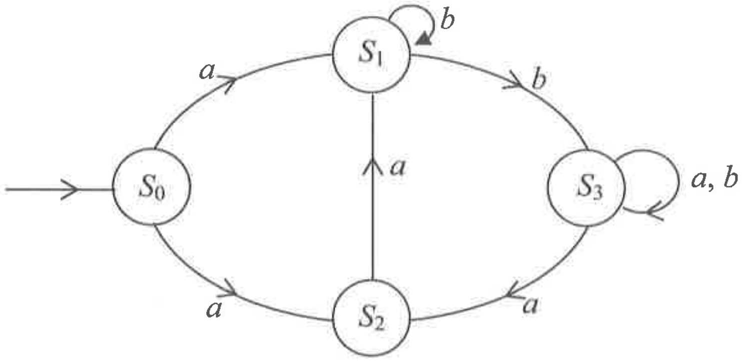
(ii) $(x + y + z + u')(x + y' + u) + (x + z + y')x'$

(f) Design a voting machine for four people in a committee in which a light glow when there is a majority vote in favour of a motion. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

3. (a) Find a deterministic automaton which accepts the same language as the non-deterministic automaton. 6



(b) Find the prime implicants and a minimal sum-of-products form for each of the following sum of products Boolean expressions: 3+3

(i) $E_1 = xyz + xyz' + x'yz' + x'y'z$

(ii) $E_2 = xyz + xyz' + xy'z + x'yz + x'y'z$.

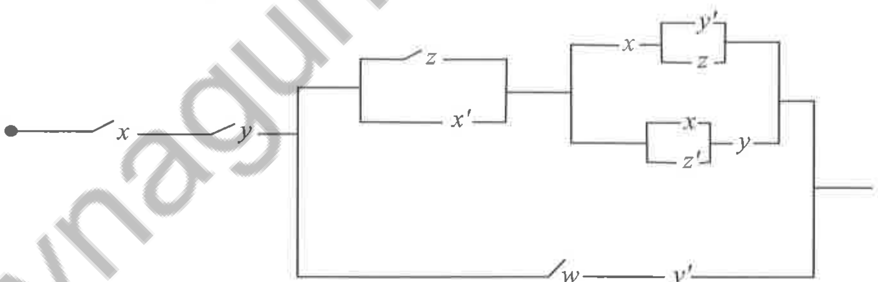
4. (a) Show that D_{30} is isomorphic to B_{31} where D_n denotes the set of all divisors of n and B_n is the set of n tuples whose members are either 0 or 1. Can we say that D_{30} is a Boolean algebra? — Justify. 2+2+2

(b) Design a DFA that accepts the following languages: 6

$L_1 = \{x \in \{0, 1\}^* : x \text{ ends in } 001\}$ and

$L_2 = \{x \in \{0, 1\}^* : x \text{ contains three consecutive 0's}\}$

5. (a) Find the Boolean expression that represents the circuit shown below: 6



Simplify if possible.

(b) Prove that a lattice is non-modular iff it contains a sublattice isomorphic to N_5. 6

6. (a) Let $G = (\{A_0, A_1, A_2, A_3\}, \{a, b\}, P, A_0)$; where P consists of $A_0 \to aA_0 \mid bA_1$, $A_1 \to aA_2 \mid aA_3$, $A_3 \to a \mid bA_1 \mid bA_3$, $A_3 \to b \mid bA_0$.
Construct an NDFA accepting $L(G)$. 6

(b) Draw the Hasse diagram for the poset $(P(S), \subseteq)$, where $P(S)$ is the power set of $S = \{1, 2, 3\}$. 6

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