'समानो मन्त्रः समितिः समानी'

# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2024

## CC14-MATHEMATICS

## PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

## (REVISED SYLLABUS 2023 / OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

*The figures in the margin indicate full marks.*

### GROUP-A

**Answer any *four* questions** $3\times4=12$

1. Solve: $p\tan x+q\tan y=\tan z$ 3

2. Show that the following system of partial differential equations are compatible and hence find the common solution: 3

$$p=(e^y+1)\cos x\,,\quad q=e^y\sin x$$

3. Form the PDE from the following by eliminating the constants: 3

$$z=(x^2+a)(y^2+b)$$

4. Find the equations of the characteristics of the P.D.E. 3

$$\sin^2 x\, z_{xx}+2\cos x\, z_{xy}-z_{yy}=0$$

5. Obtain a solution of the PDE $xp+yq=z$ representing a surface passing through the parabola $y^2=4x$, $z=1$. 3

6. Find the degree of the following PDE: 3

$$\left(\frac{\partial^2 z}{\partial x^2}\right)^2+2\frac{\partial z}{\partial y}+\sin\left(\frac{\partial z}{\partial x}\right)=x^2y$$

Write down the relation between arbitrary constants independent variables and order of a PDE.

### GROUP-B

**Answer any *four* questions** $6\times4=24$

7. Reduce the following equation to a canonical form and hence solve it: 6

$$3z_{xx}+10z_{xy}+3z_{yy}=0$$

8. Find the integral surface of the PDE $4yzp+q+2y=0$ and passing through $y^2+z^2=1$, $x+z=2$. 6

9. A particle is projected along the inner surface of a smooth vertical circle of radius $a$, its velocity at the lowest point being $\frac{1}{5}\sqrt{95ag}$. Show that it will leave the circle at an angular distance $\cos^{-1}(3/5)$ from the highest point and its velocity is $\frac{1}{5}\sqrt{15ag}$. 6

10. Solve the initial value problem: 6

$$u_{tt} = 4u_{xx}, \quad -\infty < x < \infty, \quad t > 0$$
$$u(x, 0) = x, \quad u_t(x, 0) = \cos x$$

11. Solve: $y^2(x-y)p + x^2(y-x)q = z(x^2+y^2)$ 6

12. Use Charpit's method to solve: $px + qy = pq$ 6

## GROUP-C

**Answer any *two* questions** $12 \times 2 = 24$

13.(a) A particle of mass $m$ moves under a central force $m\mu(3r^{-3} + 2a^2r^{-5})$ being projected at a distance $r = a$ with a velocity $\frac{\sqrt{5\mu}}{a}$ in a direction making an angle $\tan^{-1}\frac{1}{2}$ with the radius vector. Show that the equation of the path is $r = a\tan(\pi/4 \pm \theta)$. 6

(b) Solve by the method of separation of variables: 6

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad \text{where} \quad u(x, 0) = 6e^{-3x}$$

14.(a) Solve the boundary value problem: 6

$$u_{tt} - c^2u_{xx} = 0, \quad 0 \le x \le l, \quad t \ge 0$$

Subject to the conditions:

$$u(0, t) = u(l, t) = 0, \quad t \ge 0$$
$$u(x, 0) = 10\sin\left(\frac{\pi x}{l}\right), \quad 0 \le x \le l$$
$$u_t(x, 0) = 0$$

(b) Solve by Lagrange's method: $y^2p + x^2q = x^2y^2z^2$ 6

15.(a) A particle oscillates in a cycloid under gravity, the amplitude of motion being $b$, and period being $T$. Show that its velocity at time $t$ measured from a position of rest is $\frac{2\pi b}{T}\sin\frac{2\pi t}{T}$. 6

(b) The temperature distribution of a homogeneous thin rod, whose surface is insulated is described by the following problem: 6

$$u_t = u_{xx}, \quad 0 < x < L, \quad 0 < t < \infty$$

Subject to the conditions:

$$u(0, t) = u(L, t) = 0$$
$$u(x, 0) = f(x), \quad 0 \le x \le L$$

Find its formal solution.

16.(a) Find the integral surface of the linear PDE: $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x + y = 0$, $z = 1$. 6

(b) A particle moves in the curve $y = a\log\{\sec(x/a)\}$ in such a way that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature. 6

——×——