UNIVERSITY OF NORTH BENGAL

B.Sc. Sec 2nd Semester Examination, 2024

UMATSEC12002-MATHEMATICS

GRAPH THEORY

Time Allotted: 2 Hours   Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any *five* questions: $1 \times 5 = 5$

   (a) Give an example of a graph which is neither Eulerian nor Hamiltonian.

   (b) Define Adjacency matrix of an undirected graph.

   (c) Find the number of edges of K_7.

   (d) Define complete bipartite graph with an example.

   (e) Is it possible to draw a graph which have degree sequence 8, 8, 7, 6, 5, 4, 3, 2, 1 ? — Justify it.

   (f) For what values of m, n; $K_{m,n}$ is Hamiltonian?

   (g) What is the maximum number of edges in a simple connected graph with 10 vertices?

   (h) Give an example which has Hamiltonian cycle but not an Euler circuit.

GROUP-B

### Answer any *three* questions $5 \times 3 = 15$

2. State how adjacency matrix representation of a graph helps in checking the graph is connected or not.

3. Define binary tree. Prove that the number of internal vertices in a binary tree is one less than the number of pendant vertices.

4. Prove that a connected graph G is a tree if and only if adding an edge between any two vertices in G creates exactly one circuit.

5. Show that a connected graph G is an Euler graph if and only if all vertices of G are of even degree.

6. Show that matching M in a graph G is maximum matching if and only if G contains no M augmenting path.

GROUP-C

Answer any *two* questions
10×2 = 20

7. (a) Use Kruskal's Algorithm to find a minimum spanning tree of the weighted graph given. What is the weight of a minimum spanning tree? — 6+4
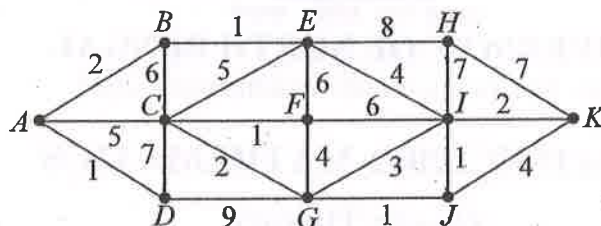

(b) Prove that a graph G is a forest if and only if $e - n + k = 0$, where $e =$ number of edges, $n =$ number of vertices of G and $k =$ number of components of G.

8. (a) Prove that, the number of odd degree vertices in a graph is always even. — 5+5

(b) Find the graph whose adjacency matrix is

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

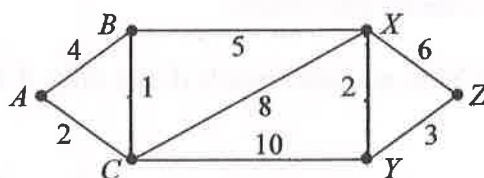9. (a) Define isomorphism of two graphs. Show that the graphs given below are isomorphic. — 5+5

(b) Using Warshall Algorithm, find the shortest path between all pairs of vertices for the following:
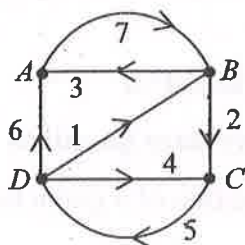
10.(a) Prove that the complete graph K_n can be expressed as the union of K bipartite graph if and only if $n \le 2^K$. — 5+5

(b) Using Dijkstra's Algorithm, find the length of the shortest path from A to Z in the following graph:

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