2024 - 2025

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Major 1st Semester Examination, 2024

MATHMAJ101-MATHEMATICS

CLASSICAL ALGEBRA AND MATRIX THEORY

Time Allotted: 2 Hours 30 Minutes | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions: $3\times4=12$

(a) Find two integers u and v satisfying $54u+24v=30$. 3

(b) Find all real values of α for which the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & \alpha \\ 5 & 7 & 1 & \alpha^2 \end{pmatrix}$ is 2. 3

(c) Find the remainder on division of 17^{17} by 8. 3

(d) Find the solution of the equation $1^z=2$. 3

(e) Check if $a^k \equiv b^k \pmod{m}$ imply $a \equiv b \pmod{m}$. 3

(f) Find the number of real roots of the equation $x^4-6x^3+10x^2-6x+1=0$. 3

GROUP-B

Answer any *four* questions $6\times4=24$

2. Prove that the equation $x^n-qx^{n-m}+r=0$ has two equal roots if 6

$$\left\{\frac{q}{n}(n-m)\right\}^n = \left\{\frac{r}{m}(n-m)\right\}^m$$

3. For distinct positive integers a, b, c, d show that 6

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}>4$$

4. Find the rank of $\begin{pmatrix} x & -1 & -1 \\ -1 & x & -1 \\ -1 & -1 & x \\ 1 & 1 & 1 \end{pmatrix}$ when $x\neq-1$ and when $x=-1$. 6

5. Find the roots of the equation $x^3-3x+1=0$. 6

6. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 1 \\ 3 & 5 & 6 \end{pmatrix}$ and show that the matrix A satisfies its characteristic equation. 6

7. State the fundamental theorem of arithmetic. Prove that the eighth power of any integer is of the form $17k$ or $17k \pm 1$ where k is a positive integer. 6

GROUP-C

Answer any *two* questions $12 \times 2 = 24$

8. (a) Solve the equation $2x^4 + 6x^3 - 3x^2 + 2 = 0$ by Ferrari's method. 6

(b) Let A and B be two real orthogonal matrices of same order and $\det A + \det B = 0$. Show that $A + B$ is singular. 6

9. (a) Solve the system of linear congruence 6

$$x \equiv 1 \pmod 3,\quad x \equiv 2 \pmod 5,\quad x \equiv 3 \pmod 7$$

(b) If P and Q are non-singular matrices, show that $\begin{pmatrix} P & 0 \\ 0 & Q \end{pmatrix}^{-1} = \begin{pmatrix} P^{-1} & 0 \\ 0 & Q^{-1} \end{pmatrix}$. 6

10.(a) If $\lambda \neq 0$ is an eigen value of a nonsingular matrix A then show that $\frac{1}{\lambda}$ is an eigen value of A^{-1}. 3

(b) If λ_1 and λ_2 are two distinct eigen values of a real symmetric 3×3 matrix and v_1, v_2 are eigen vectors corresponding to λ_1 and λ_2 then find scalar product of v_1 and v_2. 4

(c) Show that $\text{AM} \geq \text{GM} \geq \text{HM}$. When are they equal? 4+1

11.(a) Prove that any nonempty subset of the set of natural numbers has a least element. 6

(b) For what values of λ and μ the following system of equations 6

$$-4x + 2y - 9z = 2\ ,\quad 3x + 4y + z = 5\ ,\quad 3x + 4y + \lambda z = \mu$$

has (i) no solution (ii) a unique solution and (iii) infinitely many solutions over the field of rational numbers.

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