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UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2024

DSE-P1-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Symbols have their usual meanings.

The question paper contains DSE1A and DSE1B. Candidates are required to answer any *one* from the *two* DSE1 courses and they should mention it clearly on the Answer Book.

DSE1A

PROBABILITY AND STATISTICS

GROUP-A

1. Answer any *four* questions: $3\times4=12$

(a) If X be a discrete random variable having the following probability mass function: 3

X	0	1	2	3	4	5	6
$P(X=x)$	0	k	$2k$	$3k$	$4k$	$5k$	$6k$

then find (i) the constant k (ii) $P(X<5)$ and $P(X\geq4)$.

(b) Define simple and composite hypothesis. Explain them in the context of $N(m,\sigma)$ distribution. 3

(c) If X is a $\gamma(l)$ variate then compute $E(\sqrt{X})$. 3

(d) If $r(X,Y)$ be the correlation coefficient between two random variables X and Y prove that $-1\leq r\leq1$. 3

(e) A coin is tossed repeatedly until a head is obtained. If the tosses are independent and probability of head is p for each toss, find the expected number of tosses. 3

(f) The probability density function $f(x)$ of a random variable X is defined by $f(x)=\frac{1}{2}e^{-|x|}$, $-\infty<x<\infty$, find mean and variance. $1\frac{1}{2}+1\frac{1}{2}$

GROUP-B

2. Answer any *four* questions: $6\times4=24$

(a) A point P is taken at random in a line AB of length $2a$. Find the mathematical expectation of $|AP-PB|$. Also find the probability that the area exceeds $\frac{1}{2}a^2$. 3+3

(b) A random sample of size 20 from a normal population gives a sample mean of 42 and sample standard deviation 6. Test the hypothesis that the population mean is 44. State clearly the alternative hypothesis you allow for and the level of significance you adopted. 4+2

(c) State and prove Tchebycheff's inequality. 2+4

(d) If X and Y are standardized random variables and $\rho(aX+bY,\ bX+aY)=\dfrac{1+2ab}{a^2+b^2}$, find $\rho(X,Y)$, the correlation coefficient between X and Y. 6

(e) If X is a normal (m, σ) variate prove that $P(a < X < b) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right)$ and $P(|X - m| > a\sigma) = 2[1 - \Phi(a)]$, where $\Phi(x)$ denotes the standard normal distribution function. 6

(f) The joint distribution function of X and Y is given by 4+2

$$F_{XY}(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)} & , \quad x > 0,\ y > 0 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find marginal densities of X and Y and $P(X + Y \le 1)$. Are X and Y independent?

GROUP-C

3. Answer any *two* questions: 12×2 = 24

(a) (i) Let $\{X_n\}$ be a sequence of random variables such that $S_n = X_1 + X_2 + \dots + X_n$ has a finite mean M_n and finite variance B_n for all n. Then prove that 6

$$\frac{S_n - M_n}{n} \xrightarrow{\text{in } P} 0 \text{ as } n \to \infty \text{ if } \frac{B_n}{n^2} \xrightarrow{\text{in } P} 0 \text{ as } n \to \infty$$

(ii) Prove that the sum of two independent Poisson variates having parameters μ_1 and μ_2 is another Poisson variate having parameter $\mu_1 + \mu_2$. 6

(b) (i) Two random variables X and Y have least square regression lines with equations $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Find $E(X)$, $E(Y)$ and $\rho(X, Y)$. 2×3 = 6

(ii) Let p be the probability that a coin will fall 'head' in a simple toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = 3/4$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and power of the test. 3+3

(c) (i) If X and Y are two random variables having joint density function: 3+3

$$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y) & ; \quad 0 \le x < 2,\ 2 \le y < 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find $P(X + Y < 3)$ and $P(X < 1 \mid Y < 3)$.

(ii) Show that if X is a random variable having the Poisson distribution with parameter μ and $\mu \to \infty$ then the moment generating function (m.g.f.) of $Z = \frac{X - \mu}{\sqrt{\mu}}$ approximate to m.g.f. of the standard normal distribution. 6

(d) (i) Suppose X and Y be two independent random variable each having the same probability density function $f(x) = 2xe^{-x^2}$, $x > 0$. Find the pdf of $\sqrt{X^2 + Y^2}$. 6

(ii) Obtain 99% confidence interval of the population standard deviation (σ) on the basis of the data $\sum_{i=1}^{10} x_i = 620$ and $\sum_{i=1}^{10} x_i^2 = 39016$. 6

DSE1B

DIFFERENTIAL GEOMETRY

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Find the osculating plane at a point t of the curve $\vec{r} = (a\cos t, a\sin t, bt)$.

2. Check whether the surface $(z - a)^2 = xy$ is developable or not.

3. Show that the Gaussian curvature at every point of a sphere of radius a is $1/a^2$.

4. Prove that the 2nd fundamental form of a plane is zero.

5. If $\vec{r} = \vec{r}(u)$ is the equation of the curve with respect to the parameter u. Then prove that $k = \dfrac{|\dot{\vec{r}} \times \ddot{\vec{r}}|}{|\dot{\vec{r}}^3|}$.

6. Show that the parametric curves on the sphere $x = a\sin u\cos v$, $y = a\sin u\sin v$, $z = a\cos u$ forms an orthogonal system.

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Find the involutes and evolutes of the circular helix $\vec{r} = (a\cos\theta,\ a\sin\theta,\ b\theta)$.

8. Prove that first fundamental form is invariant under parametric transformation.

9. Show that the curves $u - v =$ constant are geodesic on a surface with the metric
$$(1+u^2)\,du^2 - 2uv\,du\,dv + (1+v^2)\,dv^2$$

10. If the tangent and binormal at any point on a curve make angles θ and ϕ respectively with a fixed direction then prove that
$$\frac{\sin\theta\,d\theta}{\sin\phi\,d\phi} = \frac{-k}{\tau}$$

11. Prove that if L, M, N vanish at all points on surfaces than the surface is the plane, L, M, N are second fundamental coefficient.

12. Find the Serret-Frenet approximation of the curve $\vec{r} = \vec{r}(s)$, where $s =$ arc length measured from a fixed point 0 $(s = 0)$ on the curve.

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Define Gaussian curvature. Find the value of Gaussian curvature at any point of the right helicoid $\vec{r} = (u\cos v,\ u\sin v,\ av)$. Hence show that the right helicoid is a minimal surface. 1+4+3+4

(b) Prove that the necessary and sufficient condition for a curve to be a helix is that the ratio of the curvature to the torsion is constant at all points.

14. For any curve prove that: 2+2+2+2 +2+2

(i) $\vec{r}^{I} \cdot \vec{r}^{II} = 0$

(ii) $\vec{r}^{I} \cdot \vec{r} = -k^2$

(iii) $\vec{r}^{II} \cdot \vec{r}^{III} = kk^{I}$

(iv) $\vec{r} \cdot \vec{r}^{IV} = -3kk^{I}$

(v) $\vec{r}^{II} \cdot \vec{r}^{IV} = k(k^{II} - k^3 - k\tau^2)$

(vi) $\vec{r}^{III} \cdot \vec{r}^{IV} = k^{I}k^{II} + 2k^3k^{I} + k^2\tau\tau^{I} + kk^{I}\tau^2$

15.(a) Prove that a surface is a developable surface iff the specific curvature is zero at all points. 6+6

(b) Prove that the geodesic curvature of a geodesic on a surface is zero and conversely.

16.(a) State and prove Serret-Frenet formulae. 6+3+3

(b) Find the arc length parametrization for each of the following curves.

(i) $\vec{r}(t) = 4\cos t\,\hat{i} - 4\sin t\,\hat{j}$, $t \ge 0$

(ii) $\vec{r}(t) = (t+3,\ 2t-4,\ 2t)$, $t \ge 3$

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