'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2024

CC4-MATHEMATICS

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

Answer any *four* questions $3\times4=12$

1. Find the Wronskian of e^x, $\cosh x$, $\sinh x$. Are they linearly independent? Justify your answer. 3

2. Prove that e^{x^2} is an integrating factor of the equation $(x^2+xy^4)dx+2y^3dy=0$. 3

3. Find the particular integral of the differential equation 3

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$$

4. Prove that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=[\vec{a}\vec{b}\vec{c}]^2$. 3

5. If $\vec{a}=3t\hat{i}+4t\hat{j}-t^3\hat{k}$, $\vec{b}=t^2\hat{i}-8t^3\hat{j}+3t\hat{k}$, then find $\frac{d}{dt}(\vec{a}\times\vec{b})$ at $t=1$. 3

6. Integrate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=(x^2-3y^2)\hat{i}+(y^2-2x^2)\hat{j}$ and C is closed ellipse $x=3\cos t$, $y=2\sin t$, $0\le t<2\pi$. 3

GROUP-B

Answer any *four* questions $6\times4=24$

7. Define autonomous system. Find the critical points and trajectories (direction field) of the system $\frac{dx}{dt}=6-3y$, $\frac{dy}{dt}=-12+3x^2$. 2+2+2

8. Solve the differential equation $\frac{dy}{dx}+\frac{y}{x}(\log y)=\frac{y}{x^2}(\log y)^2$. 6

9. Solve the following equation 6

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$$

10. Apply the method of variation of parameters to solve the equation: 6

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$$

11. Find div $\vec{F}$ and curl $\vec{F}$ when $\vec{F} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$. 6

12. If $\vec{F} = (5x^2 + 6y)\hat{i} - (3x + 2y^2)\hat{j} + 2xz^2\hat{k}$, then evaluate $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path C given by $x = t,\ y = t^2,\ z = t^3$. 6

GROUP-C

Answer any *two* questions $12 \times 2 = 24$

13.(a) Find an integrating factor of the equation $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$. Hence solve it. 6

(b) Solve the following system of linear differential equations: 6

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$$

$$\frac{dy}{dt} + 5x + 3y = 0$$

14.(a) Solve the equation $(D^2 - 3D + 2)y = 3x - 20\sin 2x$, by the method of undetermined coefficients. 6

(b) Solve the following equation: 6

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

15.(a) If $\vec{a} = a_1\vec{l} + a_2\vec{m} + a_3\vec{n}$, $\vec{b} = b_1\vec{l} + b_2\vec{m} + b_3\vec{n}$ and $\vec{c} = c_1\vec{l} + c_2\vec{m} + c_3\vec{n}$, then show that 6

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} [\vec{l}\,\vec{m}\,\vec{n}]$$

where $\vec{l}, \vec{m}, \vec{n}$ are three non-coplanar vectors.

(b) Find the work done by the force $\vec{F} = (0, 0, -mg)$ in moving a particle of mass m from (0, 0, 0) to (1, 1, 1) along the curve $\vec{r} = (t, t^2, t^3)$, t being a parameter. 6

16.(a) Verify that $(2x^2 + 3x)y_2 + (6x + 3)y_1 + 2y = (x + 1)e^x$ is exact and then solve it. 6

(b) A particle moves along the curve $x = t^3 + 1,\ y = t^2,\ z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $(\hat{i} + \hat{j} + 3\hat{k})$. 6

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