'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2024

CC4-MATHEMATICS

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

Answer any *four* questions $3\times4 = 12$

1. Find the Wronskian of e^x, $\cosh x$, $\sinh x$. Are they linearly independent? Justify your answer. 3

2. Find $\frac{1}{D^2+a^2}\tan ax$. 3

3. Find the particular integral of the differential equation: 3

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$$

4. Prove that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=[\vec{a}\vec{b}\vec{c}]^2$. 3

5. If $\vec{a}=3t\hat{i}+4t\hat{j}-t^3\hat{k}$, $\vec{b}=t^2\hat{i}-8t^3\hat{j}+3t\hat{k}$, then find $\frac{d}{dt}(\vec{a}\times\vec{b})$ at $t=1$. 3

6. Integrate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=(x^2-3y^2)\hat{i}+(y^2-2x^2)\hat{j}$ and C is closed ellipse $x=3\cos t$, $y=2\sin t$, $0\le t<2\pi$. 3

GROUP-B

Answer any *four* questions $6\times4 = 24$

7. Define autonomous system. Find the critical points and trajectories (direction field) of the system $\frac{dx}{dt}=6-3y$, $\frac{dy}{dt}=-12+3x^2$. 2+2+2

8. Show that 1, x, $\frac{1}{x}$ are three linearly independent solution of $x\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}=0$. Hence, find the general solution of it. 6

9. Solve the following equation. 6

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$$

10. Apply the method of variation of parameters to solve the equation: 6

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\frac{e^{-x}}{x^2}$$

11. Find div $\vec{F}$ and curl $\vec{F}$ when $\vec{F}=\text{grad}(x^3+y^3+z^3-3xyz)$. 6

12. If $\vec{F}=(5x^2+6y)\hat{i}-(3x+2y^2)\hat{j}+2xz^2\hat{k}$, then evaluate $\int_C \vec{F}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path C given by $x=t$, $y=t^2$, $z=t^3$. 6

GROUP-C

Answer any *two* questions $12\times2=24$

13.(a) Solve the equation $\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+3y=0$ near the ordinary point $x=0$. 6

(b) Solve: 6

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0$$

$$\frac{dy}{dt}+5x+3y=0$$

14.(a) Solve: $(D^2-3D+2)y=3x-20\sin 2x$, by the method of undetermined coefficients. 6

(b) Solve the following equation: 6

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$$

15.(a) If $\vec{a}=a_1\vec{l}+a_2\vec{m}+a_3\vec{n}$, $\vec{b}=b_1\vec{l}+b_2\vec{m}+b_3\vec{n}$ and $\vec{c}=c_1\vec{l}+c_2\vec{m}+c_3\vec{n}$, then show that 6

$$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}[\vec{l}\,\vec{m}\,\vec{n}]$$

where $\vec{l}, \vec{m}, \vec{n}$ are three non-coplanar vectors.

(b) Find the work done by the force $\vec{F}=(0, 0, -mg)$ in moving a particle of mass m from (0, 0, 0) to (1, 1, 1) along the curve $\vec{r}=(t, t^2, t^3)$, t being a parameter. 6

16.(a) A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+5$, where t is the time. Find the components of its velocity and acceleration at time $t=1$ in the direction $(\hat{i}+\hat{j}+3\hat{k})$. 6

(b) Solve: $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}=\frac{1}{(1-x)^2}$ 6

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