'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2024

CC12-MATHEMATICS

NUMERICAL METHODS

(REVISED SYLLABUS 2023 / OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

Answer any *five* questions from the following $1\times5=5$

1. Evaluate $\nabla^6[(1-x)(1-2x^2)(1-3x^3)]$ taking $h=2$.
2. If Π is approximated by 3.141, find the relative error.
3. Find the number of significant figure in $V_T=1.5923$ given its relative error as 1×10^{-4}.
4. Prove that $E\Delta=\Delta E$, where E and Δ are shift and forward difference operators respectively.
5. Show that the rate of convergence of Bisection method is linear.
6. Estimate the missing term in the following table:

x	0	1	2	3	4
$f(x)$	0	3	9	–	81

7. Write down the error term in Simpson $\frac{1}{3}$ rule.
8. Define diagonal dominancy and how it is useful to understand the convergency of Gauss-Seidel iteration method.

GROUP-B

Answer any *three* questions from the following $5\times3=15$

9. Evaluate $\int_0^1 \sin x\,dx$, correct upto three significant figure, considering ten equal intervals by composite Simpson's $\frac{1}{3}$ rule.
10. Show that Gauss-Seidel iteration method is applicable to solve the following system of equations and hence solve those correct to three decimal places:

$$10x_1+x_2+x_3=12$$
$$2x_1+10x_2+x_3=13$$
$$2x_1+2x_2+10x_3=14$$

11. Find a root of an equation $f(x) = x^3 - x - 1$ using Regula-Falsi method.

12. Given $\frac{dy}{dx} = 1 - y/x$ when $y(2) = 2$, compute $y(2.1)$, by Modified Euler method correct upto four decimal places, taking $h = 0.05$.

13. Deduce Newton's forward interpolation formula with Error term.

GROUP-C

Answer any *two* questions from the following $10 \times 2 = 20$

14.(a) Determine the dominant eigenvalue and corresponding eigenvectors of a matrix by power method. 5

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

(b) Compute $f(0.16)$ from the following data: 5

x	0.1	0.2	0.3	0.4
$f(x)$	1.005	1.020	1.045	1.081

15.(a) Solve by Gauss elimination method: 5

$$x + 3y + 2z = 5$$
$$2x - y + z = -1$$
$$x + 2y + 3z = 2$$

(b) Using Gauss quadrature formula, compute the value of the integral 5

$$\int_{x=0}^{0.5} \exp(-x^2)\,dx$$

16.(a) If Δ^m denotes the m^{th} order finite forward difference operator, show that 5

$$\Delta^m(1/x) = \frac{(-1)^m m!\, h^m}{x(x+h)(x+2h)\cdots\cdots(x+mh)}$$

(b) Compute truncation and rounding off errors in evaluating the finite difference approximation to the first order derivative of $f(x)$. Hence deduce that the total computation error can be minimized by choosing the step size $h = 2\sqrt{\varepsilon/M}$, where $M = \max_{t \in [x,\ x+h]} f''(t)$ and ε is the error bound of $f(x)$. 3+2

17.(a) Prove that the relative error in the product of two approximate numbers is approximately equal to the sum of their relative errors. 3

(b) Compute the error in evaluating the integral $\int_{x=0}^{1} x^2(1-x)\,dx$ taking step length equal to 0.1. 3

(c) Solve by the method of successive approximation upto 2nd order for the following initial value problem 4

$$y' = 1 + xy \quad , \quad y(0) = 1$$

——x——