'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2024

CC1-MATHEMATICS

CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Find asymptotes, if any of the curve $y=\frac{5x}{x-3}$. 3

(b) Find envelope of the curve $y=m^2x+\frac{1}{m^2}$. 3

(c) Evaluate $\int \sin^4 x\cos^2 x\,dx$. 3

(d) Through what angle must the axes be turned to remove the term x^2 from $x^2-4xy+3y^2=0$? 3

(e) Find k such that the equation $kx^2+4xy+y^2-6x-2y+2=0$ may represent a point ellipse. 3

(f) Find the length of the arc of the parabola $y^2=16x$ measured from the vertex to an extremity of the latus rectum. 3

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) Find the surface area of the solid generated by revolving the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$. 6

(b) If $y=a\cos(\log x)+b\sin(\log x)$, show that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$. 6

(c) Find the envelope of the family of ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where parameters a and b are connected by $ab=c^2$, c being a constant. 6

(d) If $ax+by$ and $cx+dy$ are changed to $a'x'+b'y'$ and $c'x'+d'y'$ respectively from rotation of axes, show that $ad-bc=a'd'-b'c'$. 6

(e) Find the equation of the sphere for which the circle $x^2+y^2+z^2+2x-4y+2z+5=0$, $x-2y+3z+1=0$ is a great circle. 6

(f) Find the general and singular solution (if it exists) of $16x^2+2p^2y-p^3x=0$, where $p=\frac{dy}{dx}$. 6

GROUP-C

3. Answer any *two* questions from the following: 12×2 = 24

(a) (i) Reduce the equation $x^2-2xy+2y^2-4x-6y+3=0$ to its canonical form and state the nature of the conic. 6

(ii) Obtain the equation of the cylinder whose generators intersect the ellipse $9x^2+3y^2=1$, $z=0$ and are parallel to the straight line with direction ratios $1,-1,1$. 6

(b) (i) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$, if $(l\cos\alpha-ep)^2+l^2\sin^2\alpha=p^2$. 6

(ii) Find all the asymptotes of the curve $4x^3-3xy^2-y^3+2x^2-xy-y^2-1=0$. 6

(c) (i) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x}$. 6

(ii) If $I_{m,n}=\int_0^{\frac{\pi}{2}}\cos^m x\sin nx\,dx$ show that $I_{m,n}=\frac{1}{m+n}+\frac{m}{m+n}I_{m-1,n-1}$ and hence deduce $I_{m,m}=\frac{1}{2^{m+1}}\left[2+\frac{2^2}{2}+\frac{2^3}{3}+\cdots\cdots+\frac{2^m}{m}\right]$. 6

(d) (i) Solve: $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$ 6

(ii) Solve: $(x^2+y^2+4)x\,dx+(x^2-y^2+9)y\,dy=0$ 6

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