



‘সমানো মনঃ সধিতিঃ সমানী’

UNIVERSITY OF NORTH BENGAL
B.Sc. Honours 5th Semester Examination, 2023

DSE-P1-MATHEMATICS
(OLD SYLLABUS 2018)

Time Allotted: 2 Hours

Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains DSE1A and DSE1B. Candidates are required to answer any *one* from the *two* DSE1 courses and they should mention it clearly on the Answer Book.

DSE1A

PROBABILITY AND STATISTICS

GROUP-A

1. Answer any *four* questions:

3×4 = 12

- (a) If two events A and B are such that $P(A+B) = \frac{3}{4}$, $P(AB) = \frac{1}{4}$, $P(\bar{A}) = \frac{2}{3}$, then find $P(\bar{A}B)$.
- (b) Show that Tchebycheff's inequality that in 2000 throws with a coin, the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$.
- (c) State weak law of large numbers.
- (d) Let T_1 and T_2 be two unbiased estimators of the parameter θ . Under what condition $aT_1 + bT_2$ will be an unbiased estimator of θ ?
- (e) Find the characteristic function of a Binomial distribution with parameters n and p .
- (f) Let X be a random variable following Poisson distribution. If $P(X=1) = P(X=2)$, find $E(X)$.

GROUP-B

2. Answer any *four* questions:

6×4 = 24

- (a) From a pack of 52 cards, an even number of cards is drawn. Show that the probability that these consist of half of red and half of black is

$$\frac{\left[\frac{52!}{(26!)^2} - 1 \right]}{(2^{51} - 1)}$$

- (b) Find the maximum likelihood estimate of the parameter λ of a continuous population having the density function

$$f(x) = \lambda x^{\lambda-1}, \quad 0 < x < 1, \quad \lambda > 0$$

- (c) (i) Prove that the second order moment of a random variable X is minimum when taken about its mean.

3+3

- (ii) If X_1, X_2, \dots, X_n be a set of mutually independent random variables having characteristic functions $\chi_1(t), \chi_2(t), \dots, \chi_n(t)$ respectively, prove that the characteristic function $\chi(t)$ of their sum S_n is given by $\chi(t) = \chi_1(t) \cdot \chi_2(t) \cdots \chi_n(t)$.
- (d) If m and μ_r denote the mean and central r -th moment of a Poisson distribution, then prove that $\mu_{r+1} = rm\mu_{r-1} + \frac{md\mu_r}{dm}$.
- (e) If $a(\neq 0), c(\neq 0), b, d$ are constants, prove that $\rho(aX + b, cY + d) = \frac{ac \rho(X, Y)}{|a||c|}$.
- (f) If X_1, X_2, \dots, X_n are mutually independent random variables and each X_i has uniform distribution over the interval (a, b) , then find the density function of the random variable U , given by
- $$U = \min\{X_1, X_2, \dots, X_n\}$$

GROUP-C

3. Answer any *two* questions: 12×2 = 24
- (a) (i) A drug is given to 10 patients, and the increments in their blood pressure were recorded to be 3, 6, -2, 4, -3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on the change of blood pressure? Test at 5% significance level, assuming the population to be normal. 7
- (ii) The joint density function of the random variable X, Y is given by 5
- $$f(x, y) = 2 \quad (0 < x < 1, 0 < y < x)$$
- Find the marginal and conditional density functions.
- Compute $P\left(\frac{1}{4} < X < \frac{3}{4} \mid Y = \frac{1}{2}\right)$.
- (b) (i) The joint probability density function of two random variable X and Y is 6
- $$f(x, y) = 8xy, \quad 0 \leq x \leq y, \quad 0 \leq y \leq 1$$
- $$= 0, \quad \text{otherwise}$$
- Examine whether X and Y are independent. Also compute $\text{var}(X)$ and $\text{var}(Y)$.
- (ii) Let X be a random variable having Poisson distribution with parameter λ . 6
- Show that the moment generating function (mgf) of $Z = \frac{X - \lambda}{\sqrt{\lambda}}$ converges to the mgf of the standard normal distribution when $\lambda \rightarrow \infty$.
- (c) (i) Let p denotes the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0: p = 0.5$ is rejected in favour of $H_1: p = 0.6$ if 10 trials result in 7 or more heads. Calculate the probabilities of type I and type II errors. 6
- (ii) If X is a continuous random variable, prove that the first absolute moment of X is minimum when taken about the median. 6
- (d) (i) A random variable X has a density function $f(x)$ given by 6
- $$f(x) = e^{-x}, \quad x \geq 0$$
- $$= 0, \quad \text{elsewhere}$$
- Show that Tchebycheff's inequality gives $P(|X - 1| \geq 2) \leq \frac{1}{4}$ and show that the actual probability is e^{-3} .

- (ii) The integers x and y are chosen at random with replacement from the nine integers 1, 2, 3, ..., 8, 9. Find the probability that $|x^2 - y^2|$ is divisible by 2. 4
- (iii) State Central limit theorem for independent and identically distributed (i.i.d) random variables with finite variance. 2

DSE1B

LINEAR PROGRAMMING

GROUP-A

1. Answer any *four* questions: 3×4 = 12

(a) Consider the following equations

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 7$$

$$2x_1 + x_2 + x_3 + 2x_4 = 3$$

Is (0, 2, 1, 0) a basic solution of the above system of equations?

(b) Solve graphically the following L.P.P.:

Minimize $Z = 4x_1 - 3x_2$

Subject to $2x_1 - x_2 \geq 4$

$$4x_1 + 3x_2 \leq 28$$

$$x_1, x_2 \geq 0$$

(c) For what value of a , the game with the following payoff matrix is strictly determinable?

		B	
		a	5
A	a	5	2
	-1	a	-8
	-2	3	a

(d) Find the optimal mixed strategies for the players A and B and the value of the game (V) with the following pay-off matrix.

$$A \begin{matrix} & \begin{matrix} B \\ -4 & 6 \\ 2 & -3 \end{matrix} \end{matrix}$$

(e) If x_1, x_2 be real, show that the set given by $X = \{(x_1, x_2) \mid x_1x_2 \leq 1, x_1, x_2 \geq 0\}$ is not a convex set.

(f) Find the dual of the following primal problem:

Minimize $Z = 3x_1 - 2x_2$

Subject to $-2x_1 - x_2 \geq -1$

$$-x_1 + 3x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

GROUP-B

2. Answer any *four* questions:

6×4 = 24

(a) An agricultural firm has 180 tons of nitrogen, 250 tons of phosphate and 220 tons of potash. The firm will be able to sell 3 : 3 : 4 mixtures of these substances at a profit of Rs. 15 per ton and 1 : 2 : 1 mixtures at a profit of Rs. 12 per ton respectively. Pose a linear programming problem to show how many tons of these mixtures should be prepared to obtain the maximum profit.

(b) Prove that a basic feasible solution to a linear programming problem corresponds to an extreme point of the convex set of feasible solutions.

(c) Use Charnes M-method to solve the L.P.P.:

$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + 5x_2 \\ \text{Subject to} \quad & 3x_1 + 4x_2 \leq 6 \\ \text{and} \quad & x_1 + 3x_2 \geq 3 \\ \text{where} \quad & x_1, x_2 \geq 0 \end{aligned}$$

(d) Find the optimal solution and corresponding minimum cost of the transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	16	18	30
O_2	12	17	20	51	40
O_3	22	28	12	32	53
b_j	22	35	25	41	

(e) Use dominance to reduce the payoff matrix and solve the game with the following pay-off matrix:

		B			
A		-5	3	1	20
		5	5	4	6
		-4	-2	0	-5

(f) Suppose that a constraint in a given L.P.P. (considered to be primal) is an equality. Prove that the corresponding dual variable is unrestricted in sign.

GROUP-C

3. Answer any *two* questions:

12×2 = 24

(a) (i) Prove that the number of basic variables in a transportation problem is at most $(m + n - 1)$, where 'm' is the number of origins and 'n' the number of destinations. 6

(ii) Consider the problem of assigning five operators to five machines. The assignment costs in rupees are given in the following table. Operator B cannot be assigned to machine 2 and operator E cannot be assigned to machine 4. Find the optimal cost of assignment and optimal assignment. 6

	1	2	3	4	5
A	8	4	2	6	1
B	0	-	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	-	5

- (b) (i) Let $(a_{ij})_{m \times n}$ be a pay-off matrix for a two person zero-sum game. Then prove that 6

$$\max_{1 \leq i \leq m} [\min_{1 \leq j \leq n} \{a_{ij}\}] \leq \min_{1 \leq j \leq n} [\max_{1 \leq i \leq m} \{a_{ij}\}]$$

- (ii) Solve the 2×4 game graphically 5

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	2	2	3	-1
	A_2	4	3	2	6

- (iii) Define convex hull. 1

- (c) (i) Use two-phase Simplex method to solve the following L.P.P.: 6

$$\begin{aligned} \text{Minimize } & Z = x_1 + x_2 \\ \text{Subject to } & 2x_1 + x_2 \geq 4 \\ & x_1 + 7x_2 \geq 7 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- (ii) Find the extreme points of the convex set of the feasible solutions of the L.P.P.: 6

$$\begin{aligned} \text{Maximize } & Z = 4x_1 + 7x_2 \\ \text{Subject to } & 2x_1 + 5x_2 \leq 40 \\ & x_1 + x_2 \leq 11 \\ \text{and } & x_1, x_2 \geq 0 \end{aligned}$$

Find also the maximum value of the objective function.

- (d) (i) $x_1 = 2, x_2 = 4$ and $x_3 = 1$ is a feasible solution to the system of equations 6

$$\begin{aligned} 2x_1 - x_2 + 2x_3 &= 2 \\ x_1 + 4x_2 &= 18 \end{aligned}$$

Reduce the feasible solution to a basic feasible one.

- (ii) Find the optimal solution of the following problem by solving its dual: 6

$$\begin{aligned} \text{Maximize } & Z = 3x_1 + 4x_2, \quad x_1, x_2 \geq 0 \\ \text{Subject to } & x_1 + x_2 \leq 10 \\ & 2x_1 + 3x_2 \leq 18 \\ & x_1 \leq 8 \\ & x_2 \leq 6 \end{aligned}$$