'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2023

CC2-MATHEMATICS

ALGEBRA

(REVISED SYLLABUS 2023 / OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions: $3\times4=12$

(a) If $p\geq q\geq 5$ and $p,\ q$ are both prime, then prove that p^2-q^2 is divisible by 24.

(b) Find $g\circ f$ if $f:\mathbb{R}\longrightarrow\mathbb{R}$ be defined by $f(x)=|x|+x,\ x\in\mathbb{R}$ and $g:\mathbb{R}\longrightarrow\mathbb{R}$ defined by $g(x)=|x|-x,\ x\in\mathbb{R}$.

(c) Use Euclidean algorithm to find the integer x and y such that

$$\gcd(72,\ 92)=72x+92y$$

(d) Let $\alpha=\begin{pmatrix}1&2&3&4&5&6&7&8&9\\3&8&7&2&9&6&5&4&1\end{pmatrix}$. Express α as a product of 2-cycles. Is α an even permutation? Also find α^{-1}.

(e) If A and B are two square matrices of the same order and A is non-singular, prove that B and ABA^{-1} have same eigen values.

(f) Find the principal value of $(-1+i)^{1+i}$.

GROUP-B

2. Answer any ***four*** questions: $6\times4=24$

(a) State Sturm's theorem. Using Strum's function to separate the roots of the equation

$$x^3+x^2-2x-1=0$$

(b) If x is real prove that

$$i\log\frac{x-i}{x+i}=\pi-2\tan^{-1}x\ ,\quad \text{if } x>0$$
$$=-\pi-2\tan^{-1}x\ ,\quad \text{if } x\leq 0$$

(c) If a,b,c,d be all positive real numbers and $s=a+b+c+d$, prove that

$$81\,abc\leq(s-a)(s-b)(s-c)(s-d)\leq\frac{81}{256}s^4$$

(d) Find the inverse of the matrix using Cayley-Hamilton theorem

$$\begin{pmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{pmatrix}$$

(e) (i) Solve $15x\equiv 9\pmod{18}$. 3+3

(ii) Prove that $3\cdot4^{n+1}\equiv 3\pmod 9$ for all positive integers n.

(f) Solve by Ferrari's method $x^4+4x^3-6x^2+20x+8=0$. 6

GROUP-C

3. Answer any *two* questions: $12\times 2=24$

(a) (i) If α, β, γ be the roots of the equation $x^3+qx+r=0$, find the equation whose roots are $\frac{\alpha+\beta}{\alpha\beta}$, $\frac{\beta+\gamma}{\beta\gamma}$, $\frac{\gamma+\alpha}{\gamma\alpha}$ and hence find out the value of $\sum\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{2}{\gamma}\right)$. 4+1

(ii) Obtain the row reduced echelon form of the matrix: 3+1

$$\begin{pmatrix} 1 & 3 & -2 & -3 \\ 2 & 1 & 4 & -1 \\ 3 & 2 & 1 & 5 \\ 1 & 3 & 5 & 4 \end{pmatrix}$$

and find the rank.

(iii) Investigate the nature of roots of the equation 3

$$x^5-x^4+8x^2-9x-15=0$$

by using Descartes' rule of signs.

(b) (i) Determine the conditions of λ and μ for which the system of equations 6

$$-4x+2y-9z=2$$
$$3x+4y+z=5$$
$$3x+4y+\lambda z=\mu$$

admits of (I) only one solution

(II) no solution

(III) many solutions.

(ii) State Cauchy-Schwartz inequality for real numbers. 2

(iii) Prove that $n!>2^n$ for all $n\geq 4$. 4

(c) (i) Let R be a relation on a set A. Define $\tau(R)=R\cup R^{-1}\cup\{(x,x)/x\in A\}$. Show that R is reflexive and symmetric. 4

(ii) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, prove that 5

$$\sum\cos^2\alpha=\sum\sin^2\alpha=\frac{3}{2}$$

(iii) If α, β, γ be the roots of equation $x^3+px^2+qx+r=0$ then find the value of $\sum(\beta+\gamma-\alpha)^3$. 3

(d) (i) For the matrix $A=\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$, find a matrix P such that $P^{-1}AP$ is a diagonal matrix. 6

(ii) Solve the equation $x^4-12x^3+48x^2-72x+35=0$ by removing the second term. 4

(iii) Stage Fundamental theorem of arithmetic. 2

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