



‘समानो मन्त्रः समितिः समानी’

UNIVERSITY OF NORTH BENGAL  
B.Sc. Honours 1st Semester Examination, 2023

CC1-MATHEMATICS

CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION  
(OLD SYLLABUS 2018)

Time Allotted: 2 Hours

Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any *four* questions from the following: 3×4 = 12
- (a) Determine the length of one arch of the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ .
- (b) Show that the curve  $y = x^3$  has a point of inflexion at  $x = 0$ .
- (c) Evaluate  $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$ .
- (d) Solve  $\log\left(\frac{dy}{dx}\right) = ax + by$ .
- (e) Find the equation of the circle on the sphere  $x^2 + y^2 + z^2 = 49$  whose centre is at the point  $(2, -1, 3)$ .
- (f) If  $I_n = \int_0^{\pi/4} \tan^n x \, dx$ , show that  $I_{n+1} + I_{n-1} = \frac{1}{n}$ .

GROUP-B

2. Answer any *four* questions from the following: 6×4 = 24
- (a) If  $y = \sin(m \cos^{-1} \sqrt{x})$ , prove that  $\lim_{x \rightarrow 0} \frac{y_{n+1}}{y_n} = \frac{4n^2 - m^2}{4n + 2}$ . 6
- (b) Obtain the reduction formula for  $\int_0^{\pi/4} \sec^n x \, dx$  where  $n(>1)$  being a positive integer. Using this find the value of  $\int_0^{\pi/4} \sec^4 x \, dx$ . 4+2

- (c) Transform the following equation to its canonical form and determine the conic represented by it 6

$$x^2 + 4xy + 4y^2 - 20x + 10y - 50 = 0$$

- (d) Find the equation of the cylinder whose generators are parallel to the straight line  $-3x = 6y = 2z$  and whose guiding curve is the ellipse  $2x^2 + y^2 = 1, z = 0$ . 6

- (e) Solve  $\frac{dy}{dx} = \sqrt{y-x}$ . 6

- (f) Show that the asymptotes of the curve  $x^2y^2 = a^2(x^2 + y^2)$  form a square whose side is of length  $2a$ . 6

### GROUP-C

3. Answer any *two* questions from the following: 12×2 = 24

- (a) (i) Find the envelope of the family of ellipses  $\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$ , where the parameters  $\alpha$  and  $\beta$  are connected by  $\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$ . 6+6

- (ii) If  $y = e^{m \sin^{-1} x}$ , prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (m^2+n^2)y_n = 0$ .

- (b) (i) Find the area in the first quadrant bounded by  $x=0, y=0$  and  $\sqrt{x} + \sqrt{y} = \sqrt{a}$ . 6+6

- (ii) Find the volume of revolution generated by the region enclosed by  $y = \sqrt{x}$  and the lines  $y=1, x=4$  about  $x$ -axis.

- (c) (i) Solve  $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$ . 6+6

- (ii) Solve  $\frac{dy}{dx} + 2xy = xy^3$ .

- (d) (i) Find equations of the generating lines of the hyperboloid  $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$ . which passes through the point  $(2, 3, -4)$ . 6+(3+3)

- (ii) Show that the distance between two points in two dimensional plane does not change under translation and rotation of co-ordinate axes.

—x—