'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2023

CC1-MATHEMATICS

CALCULUS AND GEOMETRY

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours

Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4=12$

(a) Determine the length of one arch of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$.

(b) Show that the curve $y=x^3$ has a point of inflexion at $x=0$.

(c) Evaluate $\lim\limits_{x\to0}\dfrac{xe^x-\log(1+x)}{x^2}$.

(d) Find the points on the curve $y=x^4-6x^3+13x^2-10x+5$, where the tangent is parallel to the line $y=2x$.

(e) Find the equation of the circle on the sphere $x^2+y^2+z^2=49$ whose centre is at the point $(2,-1,3)$.

(f) If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, show that $I_{n+1}+I_{n-1}=\dfrac{1}{n}$.

GROUP-B

2. Answer any ***four*** questions from the following: $6\times4=24$

(a) If $y=\sin(m\cos^{-1}\sqrt{x})$, then prove that $\lim\limits_{x\to0}\dfrac{y_{n+1}}{y_n}=\dfrac{4n^2-m^2}{4n+2}$. 6

(b) Obtain the reduction formula for $\int_0^{\pi/4}\sec^n x\,dx$ where $n\,(>1)$ being a positive integer. Using this find the value of $\int_0^{\pi/4}\sec^4 x\,dx$. 4+2

(c) Transform the following equation to canonical form and determine the conic represented by it — 6

$$x^2+4xy+4y^2-20x+10y-50=0$$

(d) Find the equation of the cylinder whose generators are parallel to the straight line $-3x=6y=2z$ and whose guiding curve is the ellipse $2x^2+y^2=1,\ z=0$. — 6

(e) PSP' is a focal chord of a conic $\frac{l}{r}=1+e\cos\theta$. Prove that the angle between the tangents at P and P' is $\tan^{-1}\left(\frac{2e\sin\alpha}{1-e^2}\right)$, where α is the angle between the chord and the major axis. — 6

(f) Show that the asymptotes of the curve $x^2y^2=a^2(x^2+y^2)$ form a square whose side is of length $2a$. — 6

GROUP-C

3. Answer any *two* questions from the following: — $12\times2=24$

(a) (i) Find the envelope of the family of ellipses $\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$, where the parameters α and β are connected by $\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}=1$. — 6+6

(ii) If $y=e^{m\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(m^2+n^2)y_n=0$. Obtain y_n for $x=0$.

(b) (i) Find the area in the first quadrant bounded by $x=0$, $y=0$ and $\sqrt{x}+\sqrt{y}=\sqrt{a}$. — 6+6

(ii) Find the volume of revolution generated by the region enclosed by $y=\sqrt{x}$ and the lines $y=1$, $x=4$ about x-axis.

(c) (i) A tangent to the parabola $y^2+4bx=0$ meets the parabola $y^2=4ax$ at P and Q. Prove that the locus of mid-point of PQ is $y^2(2a+b)=4a^2x$. — 6+6

(ii) Show that the distance between two points in two dimensional plane does not change under translation and rotation of co-ordinate axes.

(d) (i) Find equations of the generating lines of the hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ which passes through the point $(2,3,-4)$. — 6+6

(ii) A sphere of constant radius $2a$ passes through the origin O and meets the axes in A, B, C. Show that the locus of the centroid of the tetrahedron $OABC$ is the sphere $x^2+y^2+z^2=a^2$.

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