'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

CC3-MATHEMATICS

REAL ANALYSIS

(REVISED SYLLABUS 2023 AND OLD SYLLABUS 2018)

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4=12$

(a) Prove or disprove: Every bounded sequence is a Cauchy sequence. 3

(b) Show that $\lim\limits_{n\to\infty}\left(\dfrac{2n!}{(n!)^2}\right)^{1/n}=4$. 3

(c) Examine if the series $\sum\limits_{n=2}^{\infty}\dfrac{\log n}{\sqrt{n+1}}$ is convergent or not. 3

(d) Show that a finite set is a closed set. 3

(e) Check whether the set $\left\{1,-\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{1}{3},\ldots\ldots\right\}$ is open or closed. 3

(f) Prove that $\log_{10}5$ is not rational. 3

GROUP-B

Answer any *four* questions from the following $6\times4=24$

2. Prove that the union of two countable set is countable. 6

3. Test the convergence of the series $1+\dfrac{x}{1!}+\dfrac{2^2x^2}{2!}+\dfrac{3^3x^3}{3!}+\dfrac{4^4x^4}{4!}+\ldots\ldots$ for $x>0$. 6

4. Show that every infinite bounded set has a limit point. 6

5. For any two sets X and Y of $\mathbb{R}$, prove that 3+3

(a) $\text{ext}(X\cup Y)=\text{ext}(X)\cap\text{ext}(Y)$

(b) $\text{int}(X\cap Y)=\text{int}(X)\cap\text{int}(Y)$.

6. (a) Prove that $\left\{\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots\cdots+\frac{1}{2n}\right\}_{n\in\mathbb{N}}$ is a convergent sequence. 4+2

(b) Find the upper limit and lower limit of the sequence $\left\{(-1)^n\left(1+\frac{1}{2n}\right)\right\}_{n\in\mathbb{N}}$.

7. Prove that the series 6

$$1+\frac{\alpha}{\beta}+\frac{\alpha(\alpha+1)}{\beta(\beta+1)}+\frac{\alpha(\alpha+1)(\alpha+2)}{\beta(\beta+1)(\beta+2)}+\cdots\cdots$$

where α, β are positive, converges if $\beta>\alpha+1$ and diverges if $\beta\le\alpha+1$.

GROUP-C

Answer any *two* questions from the following $12\times2=24$

8. (a) Prove that a sequence $\{x_n\}$ converges to l iff both the sub-sequences $\{x_{2n}\}$ and $\{x_{2n-1}\}$ converge to l. 4

(b) Prove that $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots\cdots+\frac{1}{\sqrt{n^2+n}}\right)=1$. 4

(c) Show that every absolutely convergent series is convergent. 4

9. (a) Find the limit points of the set $S=\{\frac{2}{p}+\frac{3}{q}:\ p,q\in\mathbb{N}\}$. Is the set S closed? Is the set S open? Justify your answer. 4+1+1

(b) Prove that the closure of a set $S\subset\mathbb{R}$ is the smallest closed set containing S. 6

10. (a) If a sequence $\{a_n\}$ is bounded then show that limit inferior and limit superior of $\{a_n\}$ are both finite. 4

(b) If $a_n=\sin\frac{n\pi}{2}+\frac{(-1)^n}{n}$, $n\in\mathbb{N}$, then show that $\underline{\lim}\,a_n=-1$ and $\overline{\lim}\,a_n=1$. 4

(c) Show that the sequence $\{S_n\}$, where $S_n=1+\frac{1}{2}+\frac{1}{3}+\cdots\cdots+\frac{1}{n}$ cannot converge. 4

11. (a) Prove that if $\sum_{n=1}^{\infty}a_n$ be convergent series of positive real numbers, then $\sum_{n=1}^{\infty}a_{2n}$ is convergent. 6

(b) Prove that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\cdots\cdots$ converges for $p>1$ and diverges for $p\le1$. 6

——×——

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

CC4-MATHEMATICS

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

Answer any *four* questions from the following $3\times4=12$

1. Solve $(x+y+1)\frac{dy}{dx}=1$. 3

2. If $\vec{a},\ \vec{b},\ \vec{c}$ be three vectors, show that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}]=[\vec{a}\ \vec{b}\ \vec{c}]^2$. 3

3. State Lipschitz condition. Show that the function $f(x,y)=xy^2$ satisfies Lipschitz condition on the region $|x|\le1,\ |y|\le1$. 3

4. Show that the differential equation $x^3\frac{d^3y}{dx^3}-6x\frac{dy}{dx}+12y=0$ has three linearly independent solutions of the form $y=x^r$. 3

5. A force $3\hat{i}+\hat{k}$ acts through the point $2\hat{i}-\hat{j}+3\hat{k}$. Find the torque of the force about the point $\hat{i}+2\hat{j}-\hat{k}$. 3

6. Find the linear differential equation with real constant coefficient that is satisfied by the function. $y=9-3x+\frac{1}{6}e^{5x}$. 3

GROUP-B

Answer any *four* questions from the following $6\times4=24$

7. Solve by the method of variation of parameters: 6

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}.$$

8. Let $\vec{F}(t)=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$. Find $\int_1^2 \vec{F}(t)\times\frac{d^2\vec{F}(t)}{dt^2}\,dt$. 6

9. Solve the following differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\sin(\log x)$ 6

10. If $\vec{r}=(a\cos t)\,\hat{i}+(a\sin t)\,\hat{j}+(at\tan\alpha)\,\hat{k}$, then prove that 6

$$\left[\frac{d\vec{r}}{dt},\ \frac{d^2\vec{r}}{dt^2},\ \frac{d^3\vec{r}}{dt^3}\right]=a^3\tan\alpha$$

11. Solve the equation $(D^2-2D+1)y=xe^x$, by the method of undetermined coefficients. 6

12. Solve : $\frac{dx}{dt}-7x+y=0$ 6

$$\frac{dy}{dt}-2x-5y=0$$

GROUP-C

Answer any *two* questions of the following $12\times2=24$

13.(a) Show that the equation of the curve, whose slope at any point (x, y) is equal to $y+2x$ and which passes through the origin, is $y=2(e^x-x-1)$ 6

(b) Show that the vector field defined by $\vec{F}=2xyz^3\hat{i}+x^2z^3\hat{j}+3x^2yz^2\hat{k}$ is irrotational. Find the scalar potential u such that $\vec{F}=\text{grad}\,u$. 6

14.(a) Solve $x^2\frac{d^2y}{dx^2}-x(x+2)\frac{dy}{dx}+(x+2)y=x^3$ given that $y=x$ and $y=xe^x$ are two linearly independent solutions of the corresponding homogeneous system. 6

(b) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$ prove that $\text{curl}(f(r)\vec{r})=\vec{0}$. 6

15.(a) Solve $\frac{d^2y}{dx^2}-\frac{2}{x}\frac{dy}{dx}+\left(a^2+\frac{2}{x^2}\right)y=0$ by reducing to normal form. 6

(b) If $u=x+y+z,\ v=x^2+y^2+z^2,\ w=yz+zx+xy$, prove that $(\text{grad}\,u)\cdot\{(\text{grad}\,v)\times(\text{grad}\,w)\}=0$. 6

16.(a) Prove that $(x+y+1)^{-4}$ is an integrating factor of the equation $(2xy-y^2-y)\,dx+(2xy-x^2-x)\,dy=0$ and hence solve it. 7

(b) If $\vec{F}=zy\hat{i}+z\hat{j}+y^2x\hat{k}$, where C is the curve: $x^2+y^2=1, z=0$, then find the value of $\oint_C \vec{F}\cdot d\vec{r}$. 5

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

CC4-MATHEMATICS

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

Answer any *four* questions from the following $3\times4=12$

1. Find $\frac{1}{D^2-9}\{e^{3x}\cosh x + e^{3x}\cdot x^2 - \sin x\}$.

2. If $\vec{a},\ \vec{b},\ \vec{c}$ be three vectors, show that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$.

3. State Lipschitz condition. Show that the function $f(x,y)=xy^2$ satisfies Lipschitz condition on the region $|x|\leq 1,\ |y|\leq 1$.

4. Show that the differential equation $x^3\frac{d^3y}{dx^3}-6x\frac{dy}{dx}+12y=0$ has three linearly independent solutions of the form $y=x^r$.

5. A force $3\hat{i}+\hat{k}$ acts through the point $2\hat{i}-\hat{j}+3\hat{k}$. Find the torque of the force about the point $\hat{i}+2\hat{j}-\hat{k}$.

6. Locate and classify the singular points of the equation
$x^2(x^2-4)\frac{d^2y}{dx^2}+3x^3\frac{dy}{dx}+4y=0$.

GROUP-B

Answer any *four* questions from the following $6\times4=24$

7. Solve by the method of variation of parameters: $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$.

8. Let $\vec{F}(t)=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$. Find $\int_1^2 \vec{F}(t)\times\frac{d^2\vec{F}(t)}{dt^2}dt$.

9. Solve the following differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\cdot\sin(\log x)$.

10. If $\vec{r}=a\cos t\,\hat{i}+a\sin t\,\hat{j}+at\tan\alpha\,\hat{k}$, then prove that

$$\left[\frac{d\vec{r}}{dt},\frac{d^2\vec{r}}{dt^2},\frac{d^3\vec{r}}{dt^3}\right]=a^3\tan\alpha$$

11. Solve the equation $(D^2-2D+1)y=xe^x$ by the method of undetermined coefficients.

12. Solve : $\frac{dx}{dt}-7x+y=0$

$\frac{dy}{dt}-2x-5y=0$

GROUP-C

Answer any *two* questions of the following $12\times2=24$

13. (a) Find the power series solution of $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-xy=0$ about the point $x=0$. 6

(b) Show that the vector field defined by $\vec{F}=2xyz^3\hat{i}+x^2z^3\hat{j}+3x^2yz^2\hat{k}$ is irrotational. Find the scalar potential u such that $\vec{F}=\text{grad}\,u$. 6

14. (a) Solve $x^2\frac{d^2y}{dx^2}-x(x+2)\frac{dy}{dx}+(x+2)y=x^3$ given that $y=x$ and $y=xe^x$ are two linearly independent solutions of the corresponding homogeneous system. 6

(b) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$, prove that $\text{curl}(f(r)\vec{r})=\vec{0}$. 6

15. (a) Solve $\frac{d^2y}{dx^2}-\frac{2}{x}\frac{dy}{dx}+\left(a^2+\frac{2}{x^2}\right)y=0$ by reducing to normal form. 6

(b) If $u=x+y+z,\ v=x^2+y^2+z^2,\ w=yz+zx+xy$, prove that $(\text{grad}\,u)\cdot\{(\text{grad}\,v)\times(\text{grad}\,w)\}=0$. 6

16. (a) Solve $(D^3-D^2+3D+5)y=e^x\cos 3x$. 6

(b) If $\vec{F}=y\hat{i}+z\hat{j}+x\hat{k}$, where C is the curve: $x^2+y^2=1,\ z=0$, then find the value of $\oint_C \vec{F}\cdot d\vec{r}$. 6

——×——