UNIVERSITY OF NORTH BENGAL

B.Sc. Programme 1st Semester Examinations, 2018

DSC1-MATHEMATICS

CALCULUS AND GEOMETRY

Time Allotted: 2 Hours   Full Marks: 60

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Group- A

বিভাগ-ক

समूह—क

1. Answer any ***four*** questions from the following: $3\times4 = 12$

নিম্নলিখিত যে-কোনো *চারটি* প্রশ্নের উত্তর দাওঃ

कुनै चार प्रश्नका उत्तर देऊ –

(a) Prove that, $(\sinh x+\cosh x)^n = \sinh nx + \cosh nx$.

প্রমাণ করো যে $(\sinh x+\cosh x)^n = \sinh nx + \cosh nx$ ।

प्रमाण गर $(\sinh x+\cosh x)^n = \sinh nx + \cosh nx$

(b) Prove that the area included between the Folium of Descarte's $x^3+y^3=3axy$ and its asymptotes $x+y+a=0$ is $\frac{3}{2}a^2$.

প্রমাণ করো যে Folium of Descarte's $x^3+y^3=3axy$ এবং এর অসীমপথ $x+y+a=0$ -এর ক্ষেত্রফল $\frac{3}{2}a^2$ ।

Folium of Descarte's $x^3+y^3=3axy$ अनि यसको अनन्त स्पइकि $x+y+a=0$ भित्र समावेश भएको क्षेत्र $\frac{3}{2}a^2$ हो भनी प्रमाण गर।

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(c) State Leibnitz rule of successive differentiation. Apply it to prove that if $y^{1/m}+y^{-1/m}=2x$, then $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$.

লাইব্‌নিৎস্‌-এর ক্রমান্বয় অন্তরকলনের সূত্র (Leibnitz rule of successive differentiation) বিবৃত করো। ইহার সাহায্যে প্রমাণ করো যদি $y^{1/m}+y^{-1/m}=2x$ হয় তবে $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$ ।

क्रमिक differentiation गर्ने Leibnitz rule को उल्लेख गर। यसको प्रयोग गरेर प्रमाण गर $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$, यदि $y^{1/m}+y^{-1/m}=2x$ भए।

(d) Find the asymptotes of the following curve $x^3+y^3=3ax^2$.

$x^3+y^3=3ax^2$ বক্রটির অসীমপথ নির্ণয় করো।

वक्र $x^3+y^3=3ax^2$ को अनन्त स्पइकि निकाल।

(e) If $I_n=\int_0^{\pi/4}\tan^n\theta\,d\theta$, then prove that $n(I_{n+1}+I_{n-1})=1$.

$I_n=\int_0^{\pi/4}\tan^n\theta\,d\theta$ হইলে প্রমাণ করো $n(I_{n+1}+I_{n-1})=1$ ।

यदि $I_n=\int_0^{\pi/4}\tan^n\theta\,d\theta$, भए $n(I_{n+1}+I_{n-1})=1$ हुन्छ भनी प्रमाण गर।

(f) Discuss the characteristics of the curve $y^2(x^2-9)=x^4$ and then sketch or trace it.

$y^2(x^2-9)=x^4$ বক্রটির গাণিতিক বৈশিষ্ট্য (characteristics) নির্ণয় করো এবং বক্রটির খসড়া চিত্র (sketch) অঙ্কন করো।

वक्र $y^2(x^2-9)=x^4$ को विशेषता वर्णन गर अनि यसको स्केच बनाऊ।

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Group- B

বিভাগ - খ

समूह–ख

Answer any ***four*** questions from the following $6\times4 = 24$

নিম্নলিখিত যে-কোনো ***চারটি*** প্রশ্নের উত্তর দাও

कुनै चार प्रश्नका उत्तर देऊ

2. (a) Find the volume of the solid of revolution formed by the rotation of the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = x_1$. 4

$y^2 = 4ax$ অধিবৃত্তের x-অক্ষের সাপেক্ষে ঘূর্ণন এবং $x = x_1$ ছেদিতাংশ দ্বারা আবদ্ধ solid of revolution-এর আয়তন নির্ণয় করো।

परिवलय $y^2 = 4ax$ लाई x अक्ष को वरिपरि घुमाउँद अनि त्यसमाथी $x = x_1$, section ले घेरेको ठोस्को परिक्रमा को आयतन निकाल।

(b) Find the parametric equation for the ellipse centred at origin and intersecting axes at (4, 0), (– 4, 0); (0, 3) and (0, – 3). 2

মূলবিন্দুতে অবস্থিত কেন্দ্র বিশিষ্ট উপবৃত্ত যা অক্ষদ্বয়কে (4, 0), (– 4, 0); (0, 3), (0, – 3) বিন্দুতে ছেদ করে তার Parametric সমীকরণ নির্ণয় করো।

मूल बिन्दुमा केन्द्रित अनि intersects (4, 0), (– 4, 0); (0, 3), (0, – 3), अनि (0, – 3) भएको अण्ड वृत्त को Parametric समिकरण खोज।

3. Establish the reduction formula for $I_{m,n} = \int \sin^m x \cos^n x\, dx$, where either m or n or both are negative integers. Using it find $\int \frac{\sin^4 x}{\cos^2 x} dx$. 4+2

$I_{m,n} = \int \sin^m x \cos^n x\, dx$, যেখানে m অথবা n অথবা উভয়েই ঋণাত্মক পূর্ণসংখ্যা। $I_{m,n}$-এর সাপেক্ষে Reduct Formula নির্ণয় করো এবং এটির সাহায্যে $\int \frac{\sin^4 x}{\cos^2 x} dx$-এর মান নির্ণয় করো।

$I_{m,n} = \int \sin^m x \cos^n x\, dx$ को reduction सूत्र निकाल, जहाँ m, अनि n अथवा दुवै negative पूर्णसंख्या हरु हो। यसलाई प्रयोग गरेर $\int \frac{\sin^4 x}{\cos^2 x} dx$ को मान निर्णय गर।

4. If $y = \cos(m \sin^{-1} x)$, prove that 4+2

(i) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$

(ii) $(y_n)_0 = \begin{cases} 0, & \text{if } n \text{ be odd} \\ -m^2(2^2-m^2)(4^2-m^2)\cdots\{(n-2)^2-m^2\}, & \text{if } n \text{ be even} \end{cases}$

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$y=\cos(m\sin^{-1}x)$ হলে প্রমাণ করো -

(i) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$

(ii) $(y_n)_0=\begin{cases}0, & \text{if } n \text{ be odd}\\ -m^2(2^2-m^2)(4^2-m^2)\cdots\{(n-2)^2-m^2\}, & \text{if } n \text{ be even}\end{cases}$

यदि $y=\cos(m\sin^{-1}x)$ भए, प्रमाण गर

(क) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$

(ख) $(y_n)_0=\begin{cases}0 & , \text{ यदि } n \text{ odd भए}\\ -m^2(2^2-m^2)(4^2-m^2)\cdots\{(n-2)^2-m^2\} & \text{यदि } n \text{ even भए।}\end{cases}$

5. (a) Find the asymptotes of the cubic $x^3-2y^3+xy(2x-y)+y(x-y)+1=0$. 4

$x^3-2y^3+xy(2x-y)+y(x-y)+1=0$ ত্রিঘাত বক্রটির অসীমপথ নির্ণয় করো।

$x^3-2y^3+xy(2x-y)+y(x-y)+1=0$ को अनन्त स्पइकि निकाल।

(b) Find the envelope of the family of straight lines $A\alpha^2+B\alpha+C=0$, where α is the variable parameter and A, B, C are linear functions of x, y. 2

$A\alpha^2+B\alpha+C=0$, যেখানে α-একটি পরিবর্তনশীল চলরাশি এবং A, B, C হলো x, y-এর সরলরৈখিক অপেক্ষক (linear functions) সরলরেখা সমূহের envelope নির্ণয় করো।

सरल रेखाहरुको समूह $A\alpha^2+B\alpha+C=0$ को परिस्पइकि निकाल। α variable प्राचल अनि A, B, C, x अनि y को रेखिक फलनहरु हो।

6. Show that the equation of the circle on the sphere $x^2+y^2+z^2-2x+4y-6z+3=0$, whose centre is $(2, 3, -4)$ are $x^2+y^2+z^2-2x+4y-6z+3=0$ & $x+5y-7z-45=0$. 6

দেখাও যে, $x^2+y^2+z^2-2x+4y-6z+3=0$ গোলকের উপরিস্থিত বৃত্ত যাহার কেন্দ্র $(2, 3, -4)$ তাহার সমীকরণ $x^2+y^2+z^2-2x+4y-6z+3=0$, $x+5y-7z-45=0$।

$(2, 3, -4)$ केन्द्र भएको गोलाकार को वृत्तको समिकरण $x^2+y^2+z^2-2x+4y-6z+3=0$ अनि $x+5y-7z-45=0$ हो भनी प्रमाण गर।

7. (a) The arc of the Cardioid $r=a(1+\cos\theta)$ specified by $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$, is rotated about the line $\theta=0$. Find the area of the generated surface of revolution. 4

$r=a(1+\cos\theta)$, $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$ Cardioid -এর বৃত্তচাপটির $\theta=0$ সরলরেখার সাপেক্ষে ঘূর্ণনের ফলে উৎপন্ন surface of revolution-এর ক্ষেত্রফল বাহির করো।

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Cardioid $r=a(1+\cos\theta)$, $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$ को चाप लाई रेखा $\theta=0$ मा घुमाउँदा बनिने परिक्रमाको धरातल को क्षेत्र निर्णय गर।

(b) Show that the curve $y=\log x$ $(x>0)$ is everywhere convex upwards. 2

দেখাও যে $y=\log x$ $(x>0)$ বক্রটি সর্বত্রই convex upwards।

वक्र $y=\log x$ $(x>0)$ सबैतिर convex upwards हुन्छ भनी प्रमाण गर।

Group- C

বিভাগ – গ

समूह–ग

Answer any *two* questions from the following 12×2 = 24

নিম্নলিখিত যে-কোনো *দুটি* প্রশ্নের উত্তর দাও

कुनै दो प्रश्नका उत्तर देऊ

8. (a) If PSP' and QSQ' are two perpendicular focal chords of a conic, then prove that $\frac{1}{PS.SP'}+\frac{1}{QS.SQ'}=$ Constant. 4

যদি PSP' এবং QSQ' কোন conic–এর দুটি পরস্পর লম্ব নাভিগামী জ্যা হয় তবে প্রমাণ করো যে $\frac{1}{PS.SP'}+\frac{1}{QS.SQ'}=$ ধ্রুবক।

यदि PSP' अनि QSQ' शाङ्कवको दुई लम्ब focal chords भए $\frac{1}{PS.SP'}+\frac{1}{QS.SQ'}=$ Constant हुन्छ भनी प्रमाण गर।

(b) Find the point of inflexion on the curve $r=a\theta^{-1/2}$. 2

$r=a\theta^{-1/2}$ বক্রটির point of inflexion নির্ণয় করো।

वक्र $r=a\theta^{-1/2}$ को inflexion बिन्दु निकाल।

(c) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$. 6

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ hyperbolic paraboloid -টির পরস্পর লম্ব generate দের ছেদবিন্দুর সঞ্চারপথ নির্ণয় করো।

Hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ को लम्बकोणीय generators को चौबाटोको बिन्दुको लोकस् निकाल।

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9. (a) Find a and b in order that $\lim_{x\to 0}\frac{a\sin 2x - b\sin x}{x^3}=1$. 4

যদি $\lim_{x\to 0}\frac{a\sin 2x - b\sin x}{x^3}=1$ হয় তবে a ও b-এর মান নির্ণয় করো।

$\lim_{x\to 0}\frac{a\sin 2x - b\sin x}{x^3}=1$भए a अनि b को मान निर्णय गर।

(b) Find the angle through which the axes must be turned so that the equation $lx-my+n=0$ $(m\neq 0)$ may be reduced to the form $ay+b=0$. 2

$lx-my+n=0$ $(m\neq 0)$ সমীকরণটি ঘূর্ণনের দ্বারা $ay+b=0$ আকারে reduce করতে হলে প্রয়োজনীয় ঘূর্ণন কোণ এর মান নির্ণয় করো।

समिकरण $lx-my+n=0$ $(m\neq 0)$ लाई $ay+b=0$ रूपमा परिणत गर्नलाई अक्षहरुलाई कति मात्रको कोणमा घुमाउँनु पर्छ, निकाल।

(c) Show that the envelope of the circles whose centres lie on the rectangular hyperbola $xy=c^2$ and which pass through its centre is $(x^2+y^2)^2=16c^2xy$. 6

দেখাও যে $xy=c^2$ সমপরাবৃত্তের উপরিস্থিত কেন্দ্রবিশিষ্ট বৃত্তসমূহ যা উপরোক্ত সমপরাবৃত্তের কেন্দ্রগামী, তাহার envelope-এর সমীকরণ $(x^2+y^2)^2=16c^2xy$ ।

Rectangular hyperbola $xy=c^2$ मा केन्द्रित अनि यसको केन्द्रबाट पार हुने वृत्तको परिस्पइकि $(x^2+y^2)^2=16c^2xy$ हो भनी प्रमाण गर।

10.(a) Prove that the spheres $S_1=x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ cut the sphere $S_2=x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ in a great circle if $2(u_1u_2+v_1v_2+w_1w_2)=2r_2^2+d_1+d_2$, where r_2 is the radius of the sphere $S_2=0$. 6

যদি $S_1=x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ গোলকটি $S_2=x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ গোলককে একটি গুরুবৃত্তে (great circle) ছেদ করে তবে প্রমাণ করো যে, $2(u_1u_2+v_1v_2+w_1w_2)=2r_2^2+d_1+d_2$ যেখানে r_2 হলো $S_2=0$ গোলকটির ব্যাসার্ধ।

गोलाकार $S_1=x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ ले गोलाकार $S_2=x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ लाई ठूलो वृत्तमा काट्छ यदि $2(u_1u_2+v_1v_2+w_1w_2)=2r_2^2+d_1+d_2$ हो भनी प्रमाण गर, r_2 गोलाकार $S_2=0$ को व्यासार्ध हो।

(b) Find the length of the parabola $y^2=16x$ measured from vertex to an extremity of the latus rectum. 6

$y^2=16x$ অধিবৃত্তটির শীর্ষবিন্দু থেকে নাভিলম্বের প্রান্তবিন্দুর দৈর্ঘ্য নির্ণয় করো।

परिवलय $y^2=16x$ लाई भर्टेक्स देखी latus rectum को चरम सम्म नापिएको लम्बाई को मान निकाल।

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11.(a) Find the equation of the cylinder which intersects the curve $ax^2+by^2+cz^2=1$, $lx+my+nz=p$ and whose generators are parallel to z-axis. 6

$ax^2+by^2+cz^2=1$, $lx+my+nz=p$ বক্রোগামী cylinder যাহার generator z-অক্ষের সমান্তরাল, তাহার সমীকরণ নির্ণয় করো।

वक्र $ax^2+by^2+cz^2=1$, $lx+my+nz=p$ लाई प्रतिच्छेद गर्ने अनि जसको generators z- अक्ष संग समानन्तर छ त्यस cylinder को समिकरण निकाल।

(b) Obtain a reduction formula for $\int \cos^m x \sin nx\, dx$, m, n being positive integers and hence show that $I_{m,n}=\int_0^{\pi/2}\cos^m x\sin nx\, dx=\frac{1}{m+n}+\frac{m}{m+n}I_{m-1,n-1}$. 6

$\int \cos^m x \sin nx\, dx$ যেখানে m, n ধনাত্মক পূর্ণসংখ্যা, সমাকলটির Reduction Formula নির্ণয় করো এবং দেখাও যে $I_{m,n}=\int_0^{\pi/2}\cos^m x\sin nx\, dx=\frac{1}{m+n}+\frac{m}{m+n}I_{m-1,n-1}$।

m, n धनात्मक पूर्णसंख्या भए $\int \cos^m x \sin nx\, dx$ तो reduction सुत्र निकाल अनि देखाउनुहोस $I_{m,n}=\int_0^{\pi/2}\cos^m x\sin nx\, dx=\frac{1}{m+n}+\frac{m}{m+n}I_{m-1,n-1}$।

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UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examinations, 2018

CC1-MATHEMATICS

CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

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GROUP-A

1. Answer any ***four*** questions from the following: $3\times 4 = 12$

(a) Find a and b in order that $\lim_{x\to 0}\dfrac{a\sin 2x - b\sin x}{x^3} = 1$. 3

(b) Solve: $y(xy+2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$. 3

(c) Find the area of the region bounded by the curves $y = x^2$ and $x = y^2$. 3

(d) Obtain the equation of the sphere for which the Circle $x^2+y^2+z^2+7y-2z+2=0$; $2x+3y+4z=8$ is a great circle. 3

(e) Determine the equation of the cone whose vertex is the point (1, 2, 3) and guiding curve is $x^2+y^2+z^2=9$; $x+y+z=1$. 3

(f) Find the points of inflexion, if any, of the curve $x = (\log y)^3$. 3

GROUP-B

2. Answer any ***four*** questions from the following: $6\times 4 = 24$

(a) State and prove Leibnitz theorem for successive derivative of product of two functions. 1+5

(b) Find all the asymptotes of the curve $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$. 6

(c) Find a reduction formula for $\int \dfrac{dx}{(a+b\cos x)^n}$, n being a positive integer greater than 1, and $a^2 \neq b^2$. Hence find $\int \dfrac{dx}{(5+4\cos x)^2}$. 4+2

(d) Discuss the nature of the conic $4x^2+4xy+y^2-12x-6y+5=0$. 6

(e) A variable sphere passes through the points $(0, 0, \pm c)$ and cuts the straight lines $y = x\tan\alpha$, $z = c$; $y = -x\tan\alpha$, $z = -c$ in the points P and P' respectively. If $PP' = 2a$, a being constant, then show that the centre of the sphere lies on the circle 6

$z = 0$, $x^2 + y^2 = (a^2 - c^2)\operatorname{cosec}^2 2\alpha$.

(f) Solve the differential equation $(8p^3 - 27)x = 12p^2 y$ and investigate whether a singular solution exists or not. 6

GROUP-C

3. Answer any *two* questions from the following: $12\times 2 = 24$

(a) (i) Prove that the envelope of the family of circles passing through the origin and having centres situated on the hyperbola $x^2 - y^2 = c^2$ is $(x^2 + y^2)^2 = 4c^2(x^2 - y^2)$. 6

(ii) Reduce the equation $\sin y \dfrac{dy}{dx} = \cos x\,(2\cos y - \sin^2 x)$ to a linear equation and hence solve it. 6

(b) (i) Find the volume of the solid generated by revolting the Cardiode $r = a(1 - \cos\theta)$ about the initial line. 5

(ii) Show that the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\dfrac{l}{r} = 1 + e\cos\theta$, if $(l\cos\alpha - ep)^2 + l^2\sin^2\alpha = p^2$. 5

(iii) Define order and degree of a differential equation with examples. 2

(c) (i) Prove that the area included between the curves $x^3 - y^3 = 3axy$ and its asymptote $x + y + a = 0$ is equal to the area of the loop. 6

(ii) Solve: $\dfrac{dy}{dx} - \dfrac{\tan y}{1+x} = (1+x)e^x \sec y$. 3

(iii) Find the equation of the sphere which passes through the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ and which touches the plane $2x - 2y - z = 15$. 3

(d) (i) Find the orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2+\lambda} + \dfrac{y^2}{b^2+\lambda} = 1$, where λ is a parameter, a, b being constant. 4

(ii) Find the envelope of family of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where $a^n + b^n = c^n$, where a, b are parameter, c being a constant. 5

(iii) Find the perimeter of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. 3

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UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examinations, 2018

CC2-MATHEMATICS

ALGEBRA

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4=12$

(a) Let $r_i;\ i=1,2,\ldots k$ be real roots and $c_i;\ i=1,2,\ldots l$ be complex roots of $x^n-1=0$, $k+l=n$. Show that $\left|\sum_{i=1}^{k} r_i\right| = \left|\sum_{i=1}^{l} c_i\right|$ and $\prod_{i=1}^{n} r_i = \left(\prod_{i=1}^{l} \bar{c_i}\right) \Big/ \left(\prod_{i=1}^{l} |c_i|^2\right)$.

(b) If the matrix of a linear transformation T on $V_2(C)$ with respect to the ordered basis $B=\{(1,0),(0,1)\}$ be $\begin{bmatrix}3 & -1\\ 2 & -5\end{bmatrix}$, then find the matrix of T with respect to the basis $B'=\{(1,1),(1,-1)\}$.

(c) If $a_1, a_2, a_3, \ldots a_n$ be the n positive numbers, then show that

$$a_1+a_2+\cdots+a_n \geq \frac{n^2}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\cdots+\frac{1}{a_n}}.$$

(d) If λ be an eigen value of a non-singular matrix A, then show that λ^{-1} is an eigen value of A^{-1}.

(e) Find minimum of the polynomial: $x^4-4x^3+7x^2-6x+7$.

(f) If the function $f:R\to R$ be defined by $f(x)=x^2+1$, then find $f^{-1}(-8)$ and $f^{-1}(17)$.

GROUP-B

2. Answer any ***four*** questions from the following: $6\times4=24$

(a) Let $a,b\in R$, $a\rho b \Leftrightarrow (a-b)$ is rational. Show that ρ is an equivalence relation, every equivalence class is countable but the family of all equivalence classes is uncountable. 2+2+2

(b) Find cardinality of power set of any set. 6

(c) Determine the conditions for which the system of equations $x+y+z=b$; $2x+y+3z=b+1$; $5x+2y+9z=b^2$ has (i) only one solution, (ii) no solution, (iii) many solutions. 6

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(d) Solve: $4x^4-20x^3+33x^2-20x+4=0$. 6

(e) Show that $\left(\frac{s-a_1}{n-1}\right)^{a_1}\left(\frac{s-a_2}{n-1}\right)^{a_2}\cdots\cdots\left(\frac{s-a_n}{n-1}\right)^{a_n}\le\left(\frac{s}{n}\right)^{s}$. 6

Where $a_1, a_2, \cdots a_n$ are n positive rational numbers and $s=a_1+a_2+\cdots+a_n$.

(f) If α,β are the roots of the equation $t^2-2t+5=0$ and n is a positive integer, prove that $\frac{(a+\alpha)^n-(a+\beta)^n}{\alpha-\beta}=2^{n+1}\sin n\phi\ \operatorname{cosec}^n\phi$, where a is a real number satisfying $\frac{a+1}{2}=\cot\phi$. 6

GROUP-C

3. Answer any *two* questions from the following: $12\times2=24$

(a) (i) Show that for any real $m\times n$ matrix A the solutions of $Ax=0$, $x\in R^n$ form a subspace S of R^n. If rank of A is r then find the dimension of S. 4+3

(ii) If $\alpha,\beta,\gamma,\delta$ be roots of $x^4-3x^3+4x^2-5x+6=0$, show that $(\alpha^2+3)(\beta^2+3)(\gamma^2+3)(\delta^2+3)=57$. 5

(b) (i) If V and W be two vector spaces over a field F and V is a finite dimensional and $T:V\to W$ is a linear transformation, then show that rank of T + nullity of $T=\dim V$. 8

(ii) The matrix of linear mapping $T:R^3\to R^2$ relative to the ordered bases $\{(0,1,1),(1,0,1),(1,1,0)\}$ of R^3 and $\{(1,0),(1,1)\}$ of R^2 is $\begin{pmatrix}1&2&4\\2&1&0\end{pmatrix}$. Find the matrix of T relative to the ordered bases $\{(1,1,0),(1,0,1),(0,1,1)\}$ of R^3 and $\{(1,1),(0,1)\}$ of R^2. 4

(c) (i) Find all real λ for which the rank of the matrix $\begin{pmatrix}1&1&1&1\\1&2&-1&\lambda\\5&7&1&\lambda^2\end{pmatrix}$ is 2. 4

(ii) Show that eigenvalues of a real symmetric matrix are all real. 4

(iii) Z is a variable complex number such that $|Z|=2$. Show that the point $z+\frac{1}{z}$ lies on an ellipse of eccentricity $\frac{4}{5}$ in the complex plane. 4

(d) (i) Prove that $1^n-3^n-6^n+8^n$ is divisible by 10 for all $n\in N$. 4

(ii) Show that the number of different reflexive relations on a set of n elements is 2^{n^2-n}. 4

(iii) If a matrix A be row equivalent to a row echelon matrix having r non-zero rows, then prove that rank of $A=r$. 4

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