'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2022

CC8-MATHEMATICS

MULTIVARIATE CALCULUS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Let $f(x, y)=\begin{cases}\dfrac{x^3+y^3}{x-y} & , \quad x \neq y \\ 0 & , \quad x=y\end{cases}$ 3

   Show that $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ does not exist.

2. Find the directional derivative of $f(x, y)=x^2+y^2$ at (1, 1) in the direction of unit vector $\beta=\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$. 3

3. If $V=f(xyz)$, prove that $x\frac{\partial V}{\partial x}=y\frac{\partial V}{\partial y}=z\frac{\partial V}{\partial z}$. 3

4. Evaluate $\int\limits_0^1\int\limits_{x^2}^{x} xy\,dx\,dy$ by changing the order of integration. 3

5. Show that the vector $\vec{V}=(4xy-z^3)\hat{i}+2x^2\hat{j}-3xz^2\hat{k}$ is irrotational. 3

6. By using double integration formula find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Let $f: D\to\mathbb{R}$, $D\subseteq\mathbb{R}^2$ and $(a, b)\in D$. Let one of the partial derivatives f_x and f_y exists and the other is continuous at (a, b). Prove that f is differentiable at (a, b). 6

8. If $u=\log r$ and $r^2=x^2+y^2+z^2$, prove that 6

$$r^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=1$$

9. Prove that $\int\limits_0^1 dx\int\limits_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dy \neq \int\limits_0^1 dy\int\limits_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dx$. 6

10. State Stoke's theorem. Verify Stoke's theorem for 6

$$\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k},$$

where the surface S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

11. Evaluate $\iint(1-x-y)^{l-1}x^{m-1}y^{n-1}dx\,dy$ taken over the interior of the triangle formed by the lines $x=0,\ y=0$; $x+y=1$; where l, m, n being all positive. 6

12. Define a conservative vector field. Prove that a vector field $\vec{F}$ is conservative over a region, if and only if $\oint\vec{F}\cdot d\vec{r}$ be zero along any closed curve in the region. 1+5 = 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Show that $\iint\{2a^2-2a(x+y)-(x^2+y^2)\}\,dx\,dy=8\pi a^4$, the region of integration being the circle $x^2+y^2+2a(x+y)=2a^2$. 6

(b) Let f be a differentiable function of two independent variables u, v and u, v be differentiable functions of one independent variable x. Prove that f is a differentiable function of x and $\frac{df}{dx}=\frac{\partial f}{\partial u}\cdot\frac{du}{dx}+\frac{\partial f}{\partial v}\cdot\frac{dv}{dx}$. 6

14.(a) Let $f(x,\ y)=\begin{cases}x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}, & xy\neq 0\\ 0, & xy=0\end{cases}$ 6

Show that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

(b) Evaluate $\iint\limits_R\frac{\sqrt{a^2b^2-b^2x^2-a^2y^2}}{\sqrt{a^2b^2+b^2x^2+a^2y^2}}\,dx\,dy$ the field of integration being R, the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 6

15.(a) Use Divergence theorem to evaluate $\iint\limits_S\vec{F}\cdot d\vec{S}$ where S is the surface enclosing the cylinder $x^2+y^2=4$, $z=0$, $z=3$ and $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$. 6

(b) Apply Green's theorem in the plane to evaluate 6

$$\oint_C\{(y-\sin x)\,dx+\cos x\,dy\}$$

where C is the triangle enclosed by the lines $y=0$, $x=\pi$, $y=\frac{2x}{\pi}$.

16.(a) Prove that the necessary and sufficient condition that the vector field defined by the vector point function $\vec{F}$ with continuous derivatives be conservative is that $\operatorname{curl}\vec{F}=\vec{\nabla}\times\vec{F}=0$. 2+4 = 6

(b) Use Stoke's theorem to prove that 6

(i) curl grad $\phi=0$, where ϕ is a scalar function.

(ii) div curl $\vec{F}=0$, where F is a vector field.

——×——

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2022

CC9-MATHEMATICS

RING THEORY AND LINEAR ALGEBRA-I

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Find a basis and dimension of the subspace W of $\mathbb{R}^3$, where $W=\{(x, y, z)\in \mathbb{R}^3: x+y+z=0\}$. 3

2. Let R be a finite ring with n elements and S be a subring of R containing m elements. Prove that m is a divisor of n. 3

3. Consider the ring $\mathbb{Z}\times\mathbb{Z}$ under component-wise addition and multiplication. Show that the set $S=\{(a,\ 0):\ a\in\mathbb{Z}\}$ is a subring of $\mathbb{Z}\times\mathbb{Z}$ having unity different from that of $\mathbb{Z}\times\mathbb{Z}$. 3

4. Prove that the set $S=\{(1,0,1),(0,1,1),(1,1,0)\}$ is a basis of $\mathbb{R}^3$. 3

5. Let D be an integral domain and $a, b\in D$. If $a^5=b^5$ and $a^8=b^8$, then prove that $a=b$. 3

6. Is the ring $2\mathbb{Z}$ isomorphic to the ring $5\mathbb{Z}$? — Justify. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. (a) Determine k, so that the set $S=\{(k,1,1),(1,k,1),(1,1,k)\}$ is linearly independent in $\mathbb{R}^3$. 3

   (b) If T is linear, then $\ker T=\{\theta\}$ iff T is injective. 3

8. (a) Let T be a linear mapping on the real vector space P_4 defined by, $T(p(x)) = x\dfrac{d}{dx}(p(x))$, $p(x) \in P_4$. Determine the matrix of T relative to the standard basis of P_4. 3

(b) Find the dimension of the subspace S of $\mathbb{R}^3$ defined by, 3

$$S = \{(x, y, z) \in \mathbb{R}^3 : \quad x + 2y = z, \;\; 2x + 3z = y\}$$

9. Let R and R' be two rings and $\phi : R \to R'$ be an onto homomorphism. If I is an ideal of R, show that $\phi(I)$ is also an ideal of R'. Will this statement still be true if ϕ is any arbitrary homomorphism from R to R'? 4+2

10. Let U and W be two subspaces of a finite dimensional vector space V. Show that $\dim(U + W) = \dim(U) + \dim(W) - \dim(U \cap W)$. 6

11. Prove that a finite integral domain is a field. 6

12.(a) Let 3

$$R = \{a + b\sqrt{3} : \; a, b \in \mathbb{Z}\},$$
$$S = \{2a + 2b\sqrt{3} : \; a, b \in \mathbb{Z}\} \text{ and}$$
$$T = \{4a + 2b\sqrt{3} : \; a, b \in \mathbb{Z}\}$$

Show that T is an ideal of S, but not an ideal of R.

(b) Find the units in the integral domain $\mathbb{Z}[i]$. 3

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Prove that the ring $(\mathbb{Z}_n, +, \cdot)$ is an integral domain iff n is a prime. 4

(b) Find $\dim(U \cap V)$, where U and V are subspaces of $\mathbb{R}^4$ given by 4

$$U = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \; x_1 + x_2 + x_3 + x_4 = 0\},$$
$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \; 2x_1 + x_2 - x_3 + x_4 = 0\}$$

(c) Let R be a ring with unity and the left ideals of R are only the null ideal and R itself. Show that R is a skew field. 4

14.(a) Extend the set $\{(1, 1, 1, 1), (1, -1, 1, -1)\}$ to a basis of $\mathbb{R}^4$. 4

(b) Give an example of a subring which is not an ideal. 2

(c) The matrix of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ relative to the ordered bases $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ of $\mathbb{R}^3$ and $\{(1, 0), (1, 1)\}$ of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$. 6

Find the matrix of T relative to the ordered bases $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ of $\mathbb{R}^3$ and $\{(1, 1), (0, 1)\}$ of $\mathbb{R}^2$.

15.(a) Does there exist an epimorphism from the ring $\mathbb{Z}_{24}$ onto the ring $\mathbb{Z}_7$? 3

(b) Let I be an ideal of a ring R. Prove that if $\mathbb{R}$ is a commutative ring with unity, then so is R/I. If R has no divisor of zero, is the same necessarily true for R/I. 6

(c) Let α, β, γ be three vectors in a vector space V, so that $\alpha+\beta+\gamma=\theta$. Show that $L(\{\alpha,\ \beta\})=L(\{\beta,\ \gamma\})=L(\{\gamma,\ \alpha\})$ 3

16.(a) Find a basis and determine the dimension of the set of all 2×2 real skew symmetric matrices. 4

(b) Show that the rings $\mathbb{R}$ and $\mathbb{C}$ are not isomorphic. 2

(c) Let R be the ring of all real valued continuous functions on $[0,1]$. A mapping $\phi: R \to \mathbb{R}$ is defined by $\phi(f)=f\left(\frac{1}{2}\right)$ $\forall f \in R$. Show that ϕ is an onto homomorphism. Determine $\ker\phi$. Prove that $R/\ker\phi \simeq \mathbb{R}$. 6

——×——

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2022

CC10-MATHEMATICS

METRIC SPACES AND COMPLEX THEORY

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Prove that in any metric space (X, d) every closed sphere is a closed set. 3

(b) Show that $f(z) = |z|^2$ is nowhere differentiable except $z = 0$. 3

(c) Suppose X is a metric space and $\{x_n\}$ is a convergent sequence in X with limit α. Show that the subset $\{x_n : n \in \mathbb{N}\} \cup \{\alpha\}$ of X is compact. 3

(d) Find the value of $\int_C \frac{z^2 - 4}{z^2 + 4}\, dz$, where $C : |z - i| = 2$. 3

(e) Prove that the real line $\mathbb{R}$ is not compact. 3

(f) Show that $\int_C (z - z_0)^n\, dz = \begin{cases} 2\pi i & , \text{ if } n = -1 \\ 0 & , \text{ if } n \neq -1 \end{cases}$, 3

where C is the circle with centre z_0 and radius $r > 0$ traversed in the anti-clockwise direction.

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) Prove that a compact metric space is complete. Is the converse true? Justify your answer. 4+2 = 6

(b) Prove that the function 6

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} \quad , \quad z \neq 0$$
$$= 0 \quad , \quad z = 0$$

is continuous and that CR equations are satisfied at the origin but $f'(0)$ does not exist.

(c) (i) Show that if two connected sets are not separated, then their union is connected. 4+2 = 6

(ii) Show that every totally bounded metric space is bounded.

(d) (i) Evaluate $\int_C \frac{\cosh(\pi z)}{z(z^2+1)}dz$ using Cauchy's integral formula, where $C:|z|=2$. 3+3 = 6

(ii) Expand $\frac{z^2-1}{(z+2)(z+3)}$ in the region $|z|>3$.

(e) (i) Prove that every non-constant polynomial $p(z)=a_0+a_1z+\cdots+a_nz^n$, has at least one-zero in ℂ. Where a_j, $j=0,1,2,\cdots,n$ are complex constants and $a_n\neq 0$. 4+2 = 6

(ii) Evaluate $\int_C \frac{e^z}{z^2-2z}\,dz$, where $C:|z|=4$.

(f) Let (X,d) and (Y,d') be two metric spaces. Show that a function $f:X\to Y$ is continuous iff for any $x\in X$ and for all sequence $\{x_n\}$ converges to x in (X,d), the sequence $\{f(x_n)\}$ converges to $f(x)$ in (Y,d'). 6

GROUP-C

3. Answer any ***two*** questions from the following: 12×2 = 24

(a) (i) If $f(z)$ is differentiable in a region G and $|f(z)|$ is constant in G, then show that $f(z)$ is constant in G. 3+6+3 = 12

(ii) State and prove Cauchy's integral formula for disk.

(iii) Prove that every compact metric space is separable.

(b) (i) If $f(z)$ is an analytic function of z, show that 3+6+3 = 12

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\operatorname{Re} f(z)|^2=2|f'(z)|^2$$

(ii) Prove that every compact metric space is complete and totally bounded.

(iii) Let A be a subset of a metric space (X,d) and $A\neq\phi$. Define $d(x,A)=\inf\{d(x,a):\,a\in A\}$, $x\in X$. Show that the map $f:X\to\mathbb{R}$ defined by $f(x)=d(x,A)$ is uniformly continuous over X.

(c) (i) Prove that a necessary and sufficient condition that a function $f(z)=u(x,\,y)+iv(x,\,y)$ tend to $l=\alpha+i\beta$ as $z=x+iy$ tend to $z_0=a+ib$ is that $\lim\limits_{(x,\,y)\to(a,b)} u(x,\,y)=\alpha$ and $\lim\limits_{(x,y)\to(a,b)} v(x,\,y)=\beta$. 4+4+4 = 12

(ii) Prove that if an entire function f is bounded for all values of z. Then f is constant.

(iii) Let f be an entire function with $f(0)=1$, $f(1)=2$ and $f'(0)=0$. If there exists $M>0$ such that $|f''(z)|\leq M$ for all $z\in\mathbb{C}$, then find $f(z)$.

(d) (i) Show that the real line $(\mathbb{R},\,d)$ is connected, when d is the usual metric. 6+6 = 12

(ii) Show that a metric space is compact iff every collection of closed sets in X having finite intersection property has non-empty intersection.

——×——

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Programme 4th Semester Examination, 2022

DSC1/2/3-P4-MATHEMATICS

D. E. AND VECTOR CALCULUS

Time Allotted: 2 Hours　　　　　　　　　　　　　　　　　　　　　　Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A / বিভাগ-ক / समूह-क

Answer any *four* questions　　　　　　　　　　　　$3\times4 = 12$

যে-কোন *চারটি* প্রশ্নের উত্তর দাও

कुनै चार प्रश्नहरूको उत्तर लेख

1. Show that the solutions of the differential equation $y'' - 2y' + 2y = 0$ are linearly independent.

দেখাও যে $y'' - 2y' + 2y = 0$ অবকল সমীকরণটির সমাধানগুলি linearly independent.

विभेदक (Differential) समिकरणमा $y'' - 2y' + 2y = 0$ को समाधानहरू रेखीय रूपमा (linearly) स्वतन्त्र छ भनी प्रमाण गर।

2. Find the particular integral of the differential equation $y'' + y = \sin 2x$.

Particular Integral-বের কর $y'' + y = \sin 2x$ -অবকল সমীকরণটির জন্য।

विभेदक समिकरण $y'' + y = \sin 2x$ को विशेष integral निर्णय गर।

3. Find the Wronskian of the set $\{1,\ x,\ x^2,\ x^3,\ x^4\}$.

$\{1,\ x,\ x^2,\ x^3,\ x^4\}$ -সেটের Wronskian নির্ণয় কর।

Set $\{1,\ x,\ x^2,\ x^3,\ x^4\}$ को Wronskian निर्यण गर।

4. Find the order and degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + \sin\left(y\dfrac{dy}{dx}\right) = 0$.

$\left(\dfrac{d^2y}{dx^2}\right)^2 + \sin\left(y\dfrac{dy}{dx}\right) = 0$ -অবকল সমীকরণটি ক্রম (order) ও ঘাত (degree) নির্ণয় কর।

विभेदक समिकरण $\left(\dfrac{d^2y}{dx^2}\right)^2 + \sin\left(y\dfrac{dy}{dx}\right) = 0$ को क्रम अति डिग्री निर्यण गर।

5. If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are linearly independent, then show that $\vec{\alpha}+\vec{\beta}$, $\vec{\beta}+\vec{\gamma}$, $\vec{\gamma}+\vec{\alpha}$ are also linearly independent.

যদি $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ linearly independent হয়, তাহলে দেখাও $\vec{\alpha}+\vec{\beta}$, $\vec{\beta}+\vec{\gamma}$, $\vec{\gamma}+\vec{\alpha}$-ও linearly independent হবে।

यदि $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ रेखीय रूपमा स्वतन्त्र छ भने $\vec{\alpha}+\vec{\beta}$, $\vec{\beta}+\vec{\gamma}$, $\vec{\gamma}+\vec{\alpha}$ पनि रेखीय रूपमा स्वतन्त्र छ भनी प्रमाण गर।

6. If $\vec{\nabla}\times\vec{A}=\vec{0}$ and $\vec{\nabla}\times\vec{B}=\vec{0}$, then show that $\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{0}$.

যদি $\vec{\nabla}\times\vec{A}=\vec{0}$ এবং $\vec{\nabla}\times\vec{B}=\vec{0}$ হয়, তবে দেখাও যে, $\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{0}$ হবে।

यदि $\vec{\nabla}\times\vec{A}=\vec{0}$ अनि $\vec{\nabla}\times\vec{B}=\vec{0}$ भने प्रमाण गर $\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{0}$ ।

GROUP-B / বিভাগ-খ / समूह-ख

Answer any *four* questions 6×4 = 24

যে-কোন *চারটি* প্রশ্নের উত্তর দাও

कुनै चार प्रश्नहरूको उत्तर लेख

7. Solve the following system of linear differential equation using operator $D\equiv\frac{d}{dx}$. 6

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0 \quad ; \quad \frac{dy}{dt}+5x+3y=0$$

নিম্নের অবকল সমীকরণের জোড়টি সমাধান কর, $D\equiv\frac{d}{dx}$ অপারেটরের সাহায্যে।

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0 \quad ; \quad \frac{dy}{dt}+5x+3y=0$$

तल दिइएको विभेदक समिकरणको प्रणाली

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0 \quad ; \quad \frac{dy}{dt}+5x+3y=0$$

को समाधान निर्णय गर। (अपरेटर $D\equiv\frac{d}{dx}$ प्रयोग गर।)

8. Solve by the method of undetermined coefficients 6

$$(D^2+6D+9)y=24e^{-3x}, \quad \text{where } D\equiv\frac{d}{dx}.$$

Method of undetermined coefficient-এর সাহায্যে সমাধান করঃ

$$(D^2+6D+9)y=24e^{-3x} \quad , \quad \text{যেখানে } D\equiv\frac{d}{dx}$$

Undetermined coefficients विधि प्रयोग गरी समाधान गर :

$$(D^2+6D+9)y=24e^{-3x} \quad , \quad D\equiv\frac{d}{dx}$$

9. Solve the differential equation by the method of variation of parameters 6

$$\frac{d^2y}{dx^2}+4y=4\sec^2 2x$$

Method of variation of parameter-এর সাহায্যে সমাধান কর: $\frac{d^2y}{dx^2}+4y=4\sec^2 2x$

variation of parameters विधि द्वारा विभेदक समिकरण $\frac{d^2y}{dx^2}+4y=4\sec^2 2x$ को समाधान गर।

10. Solve: / সমাধান করঃ / समाधान गर : 6

$$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+2y=\cos(\log x)$$

11.(a) Prove that $\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{C}\times\vec{A})+\vec{C}\times(\vec{A}\times\vec{B})=\vec{0}$. 3

প্রমাণ কর যে, $\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{C}\times\vec{A})+\vec{C}\times(\vec{A}\times\vec{B})=\vec{0}$.

प्रमाण गर : $\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{C}\times\vec{A})+\vec{C}\times(\vec{A}\times\vec{B})=\vec{0}$.

(b) Suppose $\vec{A}=\hat{j}+2\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}+3\hat{k}$. Find $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$. 3

ধর $\vec{A}=\hat{j}+2\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}+3\hat{k}$, তাহলে $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$-এর মান নির্ণয় কর।

यदि $\vec{A}=\hat{j}+2\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}+3\hat{k}$ भए, $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$ को मान निर्णय गर।

12. If the position vectors of *A*, *B*, *C* are $2\hat{i}+4\hat{j}-\hat{k}$, $4\hat{i}+5\hat{j}+\hat{k}$, $3\hat{i}+6\hat{j}-3\hat{k}$ respectively, find a vector perpendicular to the plane *ABC*. 6

A, *B* ও *C*-এর position vector যথাক্রমে $2\hat{i}+4\hat{j}-\hat{k}$, $4\hat{i}+5\hat{j}+\hat{k}$ ও $3\hat{i}+6\hat{j}-3\hat{k}$, তাহলে *ABC*- তলের লম্ব ভেক্টর নির্ণয় কর।

यदि *A*, *B*, *C* भेक्टरहरूको position vector क्रमै स्णंले $2\hat{i}+4\hat{j}-\hat{k}$, $4\hat{i}+5\hat{j}+\hat{k}$, $3\hat{i}+6\hat{j}-3\hat{k}$ भए सम्तल् (plane) *ABC* मा लम्बवत भेक्टरको निर्णय गर।

GROUP-C / বিভাগ-গ / समूह-ग

Answer any *two* questions 12×2 = 24

যে-কোন *দুটি* প্রশ্নের উত্তর দাও

कुनै दुई प्रश्नहरूको उत्तर लेख

13.(a) Solve $x^2\frac{d^2y}{dx^2}-x(x+2)\frac{dy}{dx}+(x+2)y=x^3$, given that $y=x$, $y=xe^x$ are two linear independent solutions of that corresponding homogeneous equation. 6

সমাধান করঃ $x^2\frac{d^2y}{dx^2}-x(x+2)\frac{dy}{dx}+(x+2)y=x^3$, দেওয়া আছেঃ $y=x$, $y=xe^x$ দুটি linear independent সমাধান homogeneous সমীকরণটির।

$x^2\frac{d^2y}{dx^2}-x(x+2)\frac{dy}{dx}+(x+2)y=x^3$ को समाधान गर। दिइएको छ, $y=x$, $y=xe^x$ त्यस समिकारणको अनुरूप homogeneous समिकरणको रेखोय रूपमा स्वतन्त्रीत समाधानहरू हो।

(b) Find the general solutions of the differential equation. 6

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10(x+x^{-1})$$

সাধারণ সমাধান (general solution) নির্ণয় করঃ $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10(x+x^{-1})$

विभेदक समिकारण $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10(x+x^{-1})$ को सामान्य (general) समाधान निर्णय गर।

14.(a) Show that linearly independent solutions of $y''-2y'+2y=0$ are $e^x\sin x$ and $e^x\cos x$. What is the general solution? Find the solution $y(x)$ with the conditions $y(0)=2$, $y'(0)=-3$. 6

দেখাও যে, $y''-2y'+2y=0$ অবকল সমীকরণের দুটি linearly independent solution যথাক্রমে $e^x\sin x$ ও $e^x\cos x$। সমীকরণটির সাধারণ সমাধান কি ? $y(0)=2$ ও $y'(0)=-3$ শর্তে অবকল সমীকরণের সমাধান $y(x)$ নির্ণয় কর।

$y''-2y'+2y=0$ को रेखीय रूपमा स्वतन्त्र समाधानहरू $e^x\sin x$ अनि $e^x\cos x$ हो भनी पमाण गर। यसकी सामान्य समाधान के हुन्छ ? यदि $y(0)=2$, $y'(0)=-3$ सर्तहरू दिइए $y(x)$ को समाधान निर्णय गर।

(b) Show that if y_1 and y_2 be solutions of the equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x alone and $y_2=y_1z$, then $z=1+ae^{-\int\frac{Q}{y_1}dx}$ (a is an arbitrary constant). 6

যদি y_1 ও y_2 দুটি সমাধান হয় $\frac{dy}{dx}+Py=Q$ অবকল সমীকরণটির, যেখানে P ও Q শুধু x-এর অপেক্ষক, এবং যদি $y_2=y_1z$-হয়, তবে প্রমাণ কর যে $z=1+ae^{-\int\frac{Q}{y_1}dx}$ (a-একটি অনির্দিষ্ট ধ্রুবক)।

यदि y_1 अनि y_2 समिकरण $\frac{dy}{dx}+Py=Q$ को समाधान हरू भए, जहाँ P अनि Q x को भए function भए अनि $y_2=y_1z$ भए,
$z=1+ae^{-\int\frac{Q}{y_1}dx}$ भनी प्रमाण गर। (a एउटा मनमानी स्थिर arbitrary constant हो)

15.(a) Solve by the method of differentiation 6

$$\frac{dx}{dt}=7x-y \quad ; \quad \frac{dy}{dt}=2x+5y$$

Method of differentiation-এর দ্বারা সমাধান কর:

$$\frac{dx}{dt}=7x-y \quad ; \quad \frac{dy}{dt}=2x+5y$$

Differentiation विधि ले समाधान गर

$$\frac{dx}{dt}=7x-y \quad ; \quad \frac{dy}{dt}=2x+5y$$

(b) Show that $\operatorname{div}(\vec{A}\times\vec{B})=\vec{B}\cdot\operatorname{curl}\vec{A}-\vec{A}\cdot\operatorname{curl}\vec{B}$. 6

দেখাও যে, $\operatorname{div}(\vec{A}\times\vec{B})=\vec{B}\cdot\operatorname{curl}\vec{A}-\vec{A}\cdot\operatorname{curl}\vec{B}$

प्रमाण गर : $\operatorname{div}(\vec{A}\times\vec{B})=\vec{B}\cdot\operatorname{curl}\vec{A}-\vec{A}\cdot\operatorname{curl}\vec{B}$

16.(a) Evaluate: $\lim_{t\to 0}\left[\frac{\sin t}{t}\hat{i}+\frac{1-\cos t}{t}\hat{j}+e^{1-t}\hat{k}\right]$ 4

মান নির্ণয় করঃ $\lim_{t\to 0}\left[\frac{\sin t}{t}\hat{i}+\frac{1-\cos t}{t}\hat{j}+e^{1-t}\hat{k}\right]$

$\lim_{t\to 0}\left[\frac{\sin t}{t}\hat{i}+\frac{1-\cos t}{t}\hat{j}+e^{1-t}\hat{k}\right]$ को मान निर्णय गर।

(b) Find the co-ordinates of the point where the line $\vec{r}=t\hat{i}+(1+2t)\hat{j}-3t\hat{k}$ intersects the plane $3x-y-z=2$. 4

$\vec{r}=t\hat{i}+(1+2t)\hat{j}-3t\hat{k}$ -রেখাটি $3x-y-z=2$ তলে যে বিন্দুতে অন্তর্চ্ছেদ করে, তার স্থানাঙ্ক নির্ণয় কর।

रेखा $\vec{r}=t\hat{i}+(1+2t)\hat{j}-3t\hat{k}$ लो सम्नल् $3x-y-z=2$ लाई प्रतिच्छेदन् (intersect) गर्छ भने त्यम विन्दुको co-ordinates निर्णय गर।

(c) Show that $\vec{a}-3\vec{b}+5\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-4\vec{c}$ vectors are coplanar. 4

দেখাও যে $\vec{a}-3\vec{b}+5\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$ ও $-2\vec{a}+3\vec{b}-4\vec{c}$ ভেক্টরগুলি সমতলীয়।

भेक्टर हरू $\vec{a}-3\vec{b}+5\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-4\vec{c}$ coplanar छ भनी प्रमाण गर।

——×——