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UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2022

CC3-MATHEMATICS

REAL ANALYSIS

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

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GROUP-A

Answer any *four* questions from the following — 3×4 = 12

1. Prove that $\mathbb{N} \times \mathbb{N}$ is an enumerable set. — 3

2. Examine whether the sequence $\left\{1+\frac{1}{1!}+\frac{1}{2!}+ \dots\dots\dots + \frac{1}{n!}\right\}$ is a Cauchy sequence. — 3

3. When is a series of constant terms called conditionally convergent? Give an example. — 3

4. Is arbitrary union of compact sets a compact set? Justify your answer. — 3

5. Check whether the set $\{-1+\frac{1}{n},\ n\in\mathbb{N}\}$ is closed or not. — 3

6. Find $\overline{\lim} U_n$ and $\underline{\lim} U_n$, where $U_n = n^{(-1)^n}$. — 3

GROUP-B

Answer any *four* questions from the following — 6×4 = 24

7. Show that the sequence $\sqrt{a},\ \sqrt{a\sqrt{a}},\ \sqrt{a\sqrt{a\sqrt{a}}}, \dots\dots\dots\ (a>0)$ is convergent. — 6

8. (a) Give example of two distinct sets A and B such that $\operatorname{int} A = \operatorname{int} B$. — 3

   (b) Prove that the set $S = \{x\in\mathbb{R}:\ \sin x \neq 0\}$ is an open set. — 3

9. Define compact set. Prove that closed and bounded subset of real numbers is compact. — 6

10. (a) Define derived set. Obtain derived set of $\left\{\frac{(-1)^m}{m}+\frac{1}{n}:\ m,\ n\in\mathbb{N}\right\}$. — 4

    (b) Prove that $\sqrt{2}$ is not a rational number. — 2

11. Use comparison test to prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p>1$ and diverges if $p \leq 1$. 6

12.(a) Use Cauchy's criterion to prove that the sequence $\left\{1+\frac{1}{2}+\frac{1}{3}+ .... +\frac{1}{n}\right\}$ does not converge. 4

(b) Find all the sub-sequential limits of the sequence $\{\sin \frac{n\pi}{3}\}$. 2

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Let $\sum_{n=1}^{\infty} U_n$ be a positive term series such that $\lim_{n\to\infty} \sqrt[n]{U_n} = l$. Prove that the series converges if $l<1$ and diverges if $l>1$. 5

(b) If $a_n > 0 \ \forall n$. Show that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ converge or diverge together. 3

(c) Test the convergence of the series $\frac{2^2}{3^2}+\frac{2^2.4^2}{3^2.5^2}+\frac{2^2.4^2.6^2}{3^2.5^2.7^2}+ ....$. 4

14.(a) Show that the set of real numbers $\mathbb{R}$ is uncountable. 6

(b) Show that the derived set of any bounded set is also bounded set. 6

15.(a) Prove that every bounded sequence of real numbers has a convergent sub-sequence. 5

(b) Prove that the sequence $\{x_n\}$, where $x_1=\sqrt{7}$ and $x_n=\sqrt{7+x_{n-1}}$ for $n=2, 3,$ 4, ….. , is convergent. 4

(c) If $\{x_n\}$ is a sequence of positive real numbers converging to l, then show that $\lim_{n\to\infty} \sqrt[n]{x_1 x_2 \,.....\, x_n} = l$. 3

16.(a) Let $S=\{x \colon x\in\mathbb{Q}$ and $x^2<2\}$, where $\mathbb{Q}$ is the set of all rational numbers. Show that $\sup S \notin \mathbb{Q}$. 6

(b) Let $S=(0, 1]$ and $T=\{\frac{1}{n} : n\in\mathbb{N}\}$. Show that $S-T$ is an open set. 6

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UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2022

CC4-MATHEMATICS

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

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GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Verify if $\exp(x)$ and $\exp(2x)$ are independent functions. 3

2. Solve: $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$ 3

3. If $\vec{r}=\cos(nt)\hat{i}+\sin(nt)\hat{j}$, where n is a constant and t varies. Show that $\vec{r}\times\frac{d\vec{r}}{dt}=nk$. 3

4. Find the directional derivative of $\phi=xy^2z+4x^2z$ at (–1, 1, 2) along the direction $(2\hat{i}+\hat{j}-2\hat{k})$. 3

5. Find the unit tangent vector at the point where $t=2$ on the curve $x=t^2+1$, $y=4t-3$, $z=2t^2-6t$. 3

6. If $y=\exp(-x^2)$ is a solution of $xy''+\alpha y'+\beta x^3y=0$ for any two real numbers α, β then find $\alpha\beta$. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Solve: $(D^2-4D+4)y=x^2+e^x+\sin(2x)$ 6

8. Find the workdone in moving a particle around a circle in xy plane if the circle has centre at origin and radius 3 and the force field is given by $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$. 6

9. Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $r=|\vec{r}|$ and $f(r)$ is a scalar function possessing first and 2nd order derivatives prove that $\nabla^2 f(r)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$. 6

10. Show that $\frac{\partial^2 \vec{f}}{\partial x \partial y} = \frac{\partial^2 \vec{f}}{\partial y \partial x}$, where $\vec{f} = (2x^2 y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + x^2 \cos y\hat{k}$. 6

11. Find the characteristic roots of the following system and hence solve it. 6

$$\dot{x} = 3x + 2y$$
$$\dot{y} = -5x + y$$

12. Solve Euler's equation: $(x+1)^2 y'' + (x+1)y' - y = 0$ 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Find three independent solutions of $x^3 \frac{d^3 y}{dx^3} - 6x\frac{dy}{dx} + 12y = 0$. 6

(b) Show that for a differentiable function $f(r)$ one must have $\text{curl}\{f(r)\vec{r}\} = \vec{0}$, where $r = |\vec{r}|$. 6

14.(a) Prove that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ is a conservative force field. Find the scalar potential V such that $\vec{F} = \nabla V$. 6

(b) A particle p is moving on a circle of radius r with constant angular velocity $\omega = d\theta/dt$. Show that the acceleration is $-\omega^2 \vec{r}$. 6

15.(a) Solve the differential equation by the method of undetermined co-efficients 6

$$\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$$

(b) Prove that a necessary and sufficient condition for a vector $\vec{r} = \vec{f}(t)$ to have a constant direction is $\vec{f} \times \frac{d\vec{f}}{dt} = \vec{0}$. 6

16.(a) Obtain expressions for radial and transverse velocities of a moving particle in a plane and hence show that the radial and transverse accelerations are $\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$ and $\frac{1}{r}\frac{d}{dt}\left(r^2 \frac{d\theta}{dt}\right)$. 6

(b) Solve: $(x+2)^2 \frac{d^2 y}{dx^2} + (2+x)\frac{dy}{dx} + 4y = 2\sin\{2\log(x+2)\}$ 6

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UNIVERSITY OF NORTH BENGAL

B.Sc. Programme 2nd Semester Examination, 2022

DSC1/2/3-P2-MATHEMATICS

ALGEBRA

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A / বিভাগ-ক / समूह-क

Answer any *four* questions from the following $3\times4 = 12$

নিম্নলিখিত যে-কোন *চারটি* প্রশ্নের উত্তর দাও

तल दिइएको प्रश्नहरूबाट कुनै चार को उत्तर लेख

1. Find the nature of the roots of the equation $x^6+x^4+x^2+2x=0$; by using Descartes rule of signs. 3

   'Descartes' চিহ্ন বিষয়ক নিয়মের সাহায্যে $x^6+x^4+x^2+2x=0$ সমীকরণের বীজগুলির প্রকৃতি নির্ণয় কর।

   Descartes चिन्हको नियम प्रयोग गरि समिकरण $x^6+x^4+x^2+2x=0$ को मूल (root) को प्रकृतिको खोज् गर।

2. If λ be an eigen value of a real orthogonal matrix A, prove that $1/\lambda$ is also an eigen value of A. 3

   যদি একটি বাস্তব লম্ব ম্যাট্রিক্স (orthogonal matrix) A-এর আইগেন মান (eigen value) λ হয়, তবে প্রমাণ কর যে, A ম্যাট্রিক্সটির $1/\lambda$ একটি আইগেন মান (eigen value) হবে।

   यदि वास्तविक अर्थोगोनल् म्याट्रिक्स A को eigen मूल्य λ गए $1/\lambda$ पनि A को eigen मूल्यहो भनि प्रमाण गर।

3. Prove that sum of the 99th powers of the roots of the equation $x^7=1$ is 0. 3

   প্রমাণ কর যে, $x^7=1$ সমীকরণের সকল বীজগুলির 99 তম ঘাতের সমষ্টি শূন্য হবে।

   समिकरण $x^7=1$ को मूल को 99th वर्ग (powers) 0 हो भनि प्रमाण गर।

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4. Find the rank of the matrix $A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}$. 3

$A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}$ ম্যাট্রিক্সটির rank নির্ণয় কর।

म्याट्रिक्स् $A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}$ को rank निकाल।

5. Show that the mapping $f:\mathbb{R}\text{-}\{3\}\longrightarrow\mathbb{R}\text{-}\{1\}$ defined by $f(x)=\frac{x-2}{x-3}$ is bijective. 3

দেখাও যে, $f:\mathbb{R}\text{-}\{3\}\longrightarrow\mathbb{R}\text{-}\{1\}$ অপেক্ষকটি bijective, যেখানে $f(x)=\frac{x-2}{x-3}$ ।

$f(x)=\frac{x-2}{x-3}$ द्वारा परिभाषित म्यापिङ $f:\mathbb{R}\text{-}\{3\}\longrightarrow\mathbb{R}\text{-}\{1\}$ bijective हो भनी प्रमाण गर।

6. Examine if the relation ρ on $\mathbb{Z}$ is an equivalence relation, 3

where $\rho=\{(a,\, b)\in\ \mathbb{Z}\times\mathbb{Z} : 3a+4b$ is divisible by 7}.

$\rho=\{(a,\, b)\in\mathbb{Z}\times\mathbb{Z} : 3a+4b$, 7 দ্বারা বিভাজ্য}, অখণ্ড সংখ্যার সেট $\mathbb{Z}$-এর উপর সমার্থতা সম্পর্ক (equivalence relation) হবে কিনা যাচাই কর।

$\mathbb{Z}$ मा सम्बन्ध $\rho=\{(a,\, b)\in\mathbb{Z}\times\mathbb{Z} : 3a+4b$ लाई 7 ले भाग जान्छ ? समानता सम्बन्ध (equivalence relation) हो ता होइन जांच गर।

GROUP-B / বিভাগ-খ / समूह-ख

Answer any *four* questions from the following 6×4 = 24

নিম্নলিখিত যে-কোন *চারটি* প্রশ্নের উত্তর দাও

तल दिइएको प्रश्नहरूबाट कुनै चार को उत्तर लेख

7. Prove that an equivalence relation ρ on a set S determines a partition of S. Conversely, each partition of S yields an equivalence relation on S. 6

প্রমাণ কর যে, একটি সেট S-এর উপর আরোপিত সমার্থতা সম্পর্ক ρ (equivalence relation) সেট 'S' টির বিভাজন (partition) নির্ধারণ করে। বিপরীতভাবে, একটি সেট S-এর প্রতিটি বিভাজন (partition) সেট S-এর উপর আরোপিত সমার্থক সম্পর্ক (equivalence relation) প্রদান করে।

Set S मा भएको समानता सम्बन्धले S को विभाजन निर्धारण गर्छ अनि उल्टो, S मा भएको विभाजनले S मा समानता सम्बन्ध उपज गर्छ भनी प्रमाण गर।

8. (a) State the De Moivre's theorem. 2

De Moivre's উপপাদ্যটি বিবৃত কর।

De Moivre's उपपाद्य उल्लेख गर।

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(b) If $x+\frac{1}{x}=2\cos\frac{\pi}{7}$, then show that $x^7+\frac{1}{x^7}=-2$. 4

যদি $x+\frac{1}{x}=2\cos\frac{\pi}{7}$ হয়, তবে দেখাও যে, $x^7+\frac{1}{x^7}=-2$ ।

यदि $x+\frac{1}{x}=2\cos\frac{\pi}{7}$ भए प्रमाण गर $x^7+\frac{1}{x^7}=-2$ ।

9. Find the eigen values and the corresponding eigen vectors of the following matrix. 6

নিম্নলিখিত ম্যাট্রিক্সটির আইগেন মানগুলি (eigen values) এবং এই মানগুলির সম্পর্কিত আইগেন ভেক্টর (eigen values)-গুলি নির্ণয় করঃ

दिइएको म्याट्रिक्स को eigen मूल्य अनि अनुरूप eigen भेक्टर खोज् गर।

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

10. Solve the equation by Ferrari's method by $x^4-2x^2+8x-3=0$. 6

$x^4-2x^2+8x-3=0$ সমীকরণটিকে Ferrari-এর পদ্ধতিতে সমাধান কর।

Ferrari's को विधि व्दारा समिकरण $x^4-2x^2+8x-3=0$ को मान् निकाल।

11. If a, b, c be all positive real numbers and $abc=k^3$, then prove that $(1+a)(1+b)(1+c)\geq(1+k)^3$. 6

যদি a, b, c সকলেই ধনাত্মক বাস্তব সংখ্যা এবং $abc=k^3$, তাহলে প্রমাণ কর যে, $(1+a)(1+b)(1+c)\geq(1+k)^3$ ।

यदि a, b, c सबै एकारात्मक वास्तविक संख्यहर् भए अनि $abc=k^3$ प्रमाण गर $(1+a)(1+b)(1+c)\geq(1+k)^3$ ।

12. Show that the equation $\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$ represents the rectangular hyperbola $x^2-y^2=xy$. 6

দেখাও যে, $\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$ সমীকরণটি $x^2-y^2=xy$ আয়তক্ষেত্রাকার অধিবৃত্তকে (rectangular hyperbola) উপস্থাপনা করে।

समिकरण $\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$ ले आयताकार हाइपरबोला $x^2-y^2=xy$ प्रतिनिधित्व गर्छ भनी प्रमाण गर।

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GROUP-C / বিভাগ-গ / समूह-ग

Answer any *two* questions from the following 12×2 = 24

নিম্নলিখিত যে-কোন *দুটি* প্রশ্নের উত্তর দাও

तल दिइएको प्रश्नहरूबाट कुनै दुईको उत्तर लेख

13.(a) Determine k and solve the equation if the roots are in A.P. 6

$$x^4 - 8x^3 + kx^2 + 8x - 15 = 0$$

k-এর মান নির্ণয় কর এবং নিম্নলিখিত সমীকরণটির সমাধান কর যদি সমীকরণটির বীজগুলি সমান্তর প্রগতিতে (A.P.) থাকে $x^4 - 8x^3 + kx^2 + 8x - 15 = 0$ ।

समिकरण $x^4 - 8x^3 + kx^2 + 8x - 15 = 0$ बाट k को मान निकाल अनि यदि मूलहरू A.P. मा भए त्यसको समाधान पनि गर।

(b) Use De Moivre's theorem to prove that 6

De Moivre's উপপাদ্য প্রয়োগ করে প্রমাণ করঃ

De Moivre's को उपपाद्य प्रयोग गरी प्रमाण गर

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$

14.(a) Let $f : A \to B$ and $g : B \to C$ be both bijective mappings. Then prove that the mapping $g \circ f : A \to C$ is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. 6

ধরা যাক, $f : A \to B$ এবং $g : B \to C$ উভয়ই bijective অপেক্ষক। তাহলে প্রমাণ কর যে, $g \circ f : A \to C$ একটি বিপরীত অপেক্ষক (invertible mapping) হবে এবং $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ ।

यदि $f : A \to B$ अनि $g : B \to C$ दुवै bijective म्यापिक भए प्रमाण गर, कि म्यापिङ $g \circ f : A \to C$ invertible हो भनी अनि $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ ।

(b) (i) If $a \equiv b \pmod{m}$ then $a^n \equiv b^n \pmod{m}$ for all integer n. Is the converse of this statement true? Justify your answer. 3+3

n-এর সকল পূর্ণসংখ্যার জন্য যদি $a \equiv b \pmod{m}$ হয়, তাহলে $a^n \equiv b^n \pmod{m}$ হবে। বিপরীত বিবৃতিটি কী সত্য ? তোমার উত্তরের যর্থাথতা যাচাই কর।

यदि $a \equiv b \pmod{m}$ भए प्रमाण गर $a^n \equiv b^n \pmod{m}$ सबै पूर्णांक n को लागी। माथिको वाक्यको उल्टो के सत्य हो ? आफ्नो उत्तरको न्यायोचित गर।

(ii) Prove that if $ax \equiv ay \pmod{m}$ and a is prime to m then $x \equiv y \pmod{m}$.

যদি $ax \equiv ay \pmod{m}$ এবং a ও m পরস্পর মৌলিক সংখ্যা হয়, তবে প্রমাণ কর যে, $x \equiv y \pmod{m}$ ।

यदि $ax \equiv ay \pmod{m}$ अनि a र m prove भए प्रमाण गर $x \equiv y \pmod{m}$ ।

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15.(a) Determine the condition for which the system of equation has (i) unique solution, (ii) no solution, (iii) many solutions. 6

নিম্নলিখিত সমীকরণ সমূহের

(ক) একটি মাত্র সমাধান আছে (খ) কোন সমাধান নেই (গ) একাধিক সমাধান আছে ক্ষেত্রে শর্তগুলি উল্লেখ কর।

निम्नलिखित समिकरणहरूको प्रणाली

(i) अद्वितीय (unique) समाधान, (ii) समाधान छैन, (iii) थुप्रै समाधान, छ भने सर्तहरू निर्धारण गर।

$$x+y+z=b$$
$$2x+y+3z=b+1$$
$$5x+2y+az=b^2$$

(b) Solve the equation by Cardan's method: $x^3-12x+65=0$ 6

$x^3-12x+65=0$ সমীকরণটিকে Cardan-এর পদ্ধতিতে সমাধান কর।

Cardan को विधि द्वारा समाधान निकालः $x^3-12x+65=0$

16.(a) If x, y, z are positive real numbers and $x+y+z=1$. Prove that 6

$$8xyz \le (1-x)(1-y)(1-z) \le 8/27$$

যদি x, y, z তিনটি ধনাত্মক বাস্তব সংখ্যা এবং $x+y+z=1$ হয়, তবে প্রমাণ কর যে,
$8xyz \le (1-x)(1-y)(1-z) \le 8/27$।

यदि x, y, z सकारात्मक वास्तविक संख्याहरू हो अनि $x+y+z=1$ हो भने, $8xyz \le (1-x)(1-y)(1-z) \le 8/27$ हुन्छ भनी प्रमाण गर।

(b) Using Cayley-Hamilton theorem find A^{50} if $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$. 6

Cayley-Hamilton উপপাদ্যটি ব্যবহার করে A^{50}-এর মানটি নির্ণয় কর যেখানে, $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$।

यदि $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ हो भने, Cayley-Hamilton उपपाद्य प्रयोग गरी A^{50} खोज।

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'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2022

GE1-P2-MATHEMATICS

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

The question paper contains MATHGE-I, MATHGE-II, MATHGE-III, MATHGE-IV & MATHGE-V.
The candidates are required to answer any *one* from the *five* courses.
Candidates should mention it clearly on the Answer Book.

MATHGE-I

CAL. GEO AND DE.

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Evaluate: $\int_0^{\pi/4} \tan^5 x\,dx$. 3

2. Find the equation of the curve $3x^2+3y^2+6x-18y-14=0$ referred to parallel axes through the point (–1, 3). 3

3. Determine the concavity and the inflexion points of $f(x)=x^3+3x^2-9x+8$. 3

4. Transform the following equation into Cartesian form $r=2\sin 3\theta$. 3

5. Solve: $(2x\cos y+3x^2y)\,dx+(x^3-x^2\sin y-y)\,dy=0$; $y(0)=2$ 3

6. Find the asymptote of the curve $x^2y^2=a^2(x^2+y^2)$. 3

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GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Find the trace of $y^2(2a-x)=x^3$. 6

8. Show that $\int \tan^2 x\sec^4 x\,dx=\frac{1}{5}\tan^5 x+\frac{1}{3}\tan^3 x$. 6

9. If $y=e^{3\sin^{-1}x}$, then show that $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-(n^2+9)\,y_n=0$. 6

10. Locate the vertex and focus of the parabola $x^2 - 4x - 12y - 15 = 0$, write the equation of the directrix axis and tangent at the vertex. 6

11. Find the envelope of $x^2 \sin\alpha + y^2 \cos\alpha = a^2$, where α is a parameter. 6

12. Solve: $xy - \dfrac{dy}{dx} = y^3 e^{-x^2}$ 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$, $|x| < 1$, show that $(1-x^2)y_{n+2} - (2n+3)x\,y_{n+1} - (n+1)^2 y_n = 0$. 6

(b) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \sin nx\, dx$, then show that $I_{m,n} = \dfrac{1}{m+n} + \dfrac{m}{m+n} I_{m-1,\, n-1}$. 6

14.(a) Determine the value of α and β for which $\lim\limits_{x\to 0} \dfrac{\sin 3x + \alpha \sin 2x + \beta \sin x}{x^5}$ exists and find the limit. 6

(b) Show that the perpendicular from the origin on the generators of the paraboloid $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = \dfrac{2z}{c}$ lie on the cone $\left(\dfrac{x}{a} - \dfrac{y}{b}\right)(ax - by) + 2z^2 = 0$. 6

15.(a) Solve: $\dfrac{dy}{dx} + x \sin xy = e^x y^n$ 6

(b) Reduce the equation $\sin y \dfrac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$ to a linear equation and hence solve it. 6

16.(a) Reduce the equation $x^2 + y^2 + z^2 - 2xy - 2yz + 2zx + x - 4y + z + 1 = 0$ to its canonical form and determine the type of the quadric represented by it. 6

(b) Determine the concavity and the inflection points of the function $f(x) = 3x^4 - 4x^2 + 1$. 6

MATHGE-II

ALGEBRA

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. If k be a positive integer then prove that $\gcd(ka, kb) = k \gcd(a, b)$.

2. Determine k so that the set S is linearly independent in $\mathbb{R}^3$; where $S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$.

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3. If a, b, c be three positive real numbers, prove that $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 3$.

4. If λ be an eigenvalue of $A\in M_n(\mathbb{R})$, prove that λ^m is an eigenvalue of A^m.

5. Find $\operatorname{mod} z$ and $\arg z$, where $z=i^i$.

6. Solve the equation $x^4-x^3+2x^2-2x+4=0$, one root being $1+i$.

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. (a) Determine the rank of the matrix $\begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 2 & 6 & 9 \\ 1 & 1 & 2 & 6 \end{pmatrix}$. 3

(b) Consider a matrix A whose eigenvalues are 1, –1 and 3. Then find trace (A^3-3A^2). 3

8. (a) If n be a positive integer, prove that $\frac{1}{\sqrt{4n+1}}<\frac{3\cdot 7\cdot 11\cdot \ldots\ldots \cdot(4n-1)}{5\cdot 9\cdot 13\cdot \ldots\ldots \cdot(4n+1)}<\sqrt{\frac{3}{4n+3}}$. 3

(b) If α be a multiple root of order 3 of the equation $x^4+bx^2+cx+d=0$, $(d\neq 0)$. Show that $\alpha=-\frac{8d}{3c}$. 3

9. (a) Prove that $3^{2n}-8n-1$ is divisible by 64 for any non-negative integer n. 3

(b) Consider the function $f:\mathbb{R}\longrightarrow(-1,1)$, defined by $f(x)=\frac{x}{1+|x|}$ for all $x\in\mathbb{R}$. Prove that f is bijective. 3

10.(a) Let $f:A\to B$ and $g:B\to C$ be two functions. Then prove that if $g\circ f$ is bijective then f is injective and g is surjective. 3

(b) What is the residue when 11^{40} is divided by 8? 3

11.(a) Find the product of all the values of $(1+i)^{4/5}$. 4

(b) State the Cauchy-Schwartz inequality. 2

12.(a) For what values of 'a' the following system of equation is consistent? 4

$$x-y+z=1$$
$$x+2y+4z=a$$
$$x+4y+6z=a^2$$

(b) Give an example of a binary relation which is reflexive and transitive but not symmetric. 2

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GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) State and prove the fundamental theorem of equivalence relation. 6

(b) Let n be a positive integer and $\mathbb{Z}_n$ denote the set of all congruence classes of $\mathbb{Z}$ modulo n. Prove that the number of elements of $\mathbb{Z}_n$ is finite. 6

14.(a) Prove the following identities: 3+3

(i) $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

(ii) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(b) Let n be a positive integer and a, b and c are integers such that $a \neq 0$. Then prove that $ab \equiv ac \pmod{n}$ if and only if $b \equiv c \left(\bmod \dfrac{n}{\gcd(a, n)}\right)$. 6

15.(a) State Cayley-Hamilton theorem for matrices. Use it to find A^{-1} and A^{50}, where $A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$. 3+3

(b) Find the rank of the following matrices of order n: 6

(i) Nilpotent matrix

(ii) Idempotent matrix and

(iii) Involuntary matrix.

16.(a) Consider the mapping $f : \mathbb{Z}_0^+ \times \mathbb{Z} \longrightarrow \mathbb{Z}$, defined by $f(m, n) = 2^m(2n+1)$ for all $(m, n) \in \mathbb{Z}_0^+ \times \mathbb{Z}$. Prove that f is injective but not surjective. Here $\mathbb{Z}_0^+$ denotes the set of all non-negative integers. 4

(b) Prove that composition of mappings is associative. Show by a counter example that composition of mapping is not commutative. 2+3

(c) Apply Descarte's rule of signs to find the positive and negative roots of the following equation: 3

$$4x^3 - 8x^2 - 19x + 26 = 0$$

MATHGE-III

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Show that $f(t, x) = \dfrac{e^{-x}}{1+t^2}$ defined for $0 < x < p$, $0 < t < N$, where N is a positive integer, satisfies Lipschitz condition with Lipschitz constant $K = p$. 3

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2. Find the Wronskian of $\{1-x,\ 1+x,\ 1-3x\}$. 3

3. Examine continuity of the vector valued function $\vec{r}=t^3\hat{i}+e^t\hat{j}+\frac{1}{t+3}\hat{k}$ at $t=-3$. 3

4. Find the directional derivative of $\phi = xy^2z+4x^2z$ at (–1, 1, 2) in the direction $2\hat{i}+\hat{j}-2\hat{k}$. 3

5. Find the particular integral of $\frac{d^2y}{dx^2}+4y=\sin 2x$. 3

6. Solve: $\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=0$, with $x(0)=0$, $\frac{dx(0)}{dt}=0$. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. If y_1 and y_2 are solutions of $y''+x^2y'+(1-x)y=0$ such that $y_1(0)=0$, $y_2(0)=1$, $y_1'(0)=1$, $y_2'(0)=-1$, then find the Wronskian $W(y_1,\ y_2)$. 6

8. Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\frac{e^{-x}}{x^2}$. 6

9. Solve: $(D^3-D^2-6D)\,y=(1+x+x^2)e^x$, where $D\equiv\frac{d}{dx}$. 6

10. Solve: 6

$$4x'+9y'+44x+49y=t$$
$$3x'+7y'+34x+38y=e^t$$

11. If $\vec{A}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$, then evaluate $\int_C \vec{A}.d\vec{r}$ along the curve C given by $x=t^2+1,\ y=2t^2,\ z=t^3$ from $t=1$ to $t=2$. 6

12. Show that the vector field given by $\vec{A}=(y^2+z^3)\hat{i}+(2xy-5z)\hat{j}+(3xz^2-5y)\hat{k}$ is conservative and find the scalar point function for the field. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Find the general solution of $t^2y''-3ty'+7y=0$, $t>0$. 6

(b) Solve: $(D-1)^2(D^2+1)\,y=e^x+\sin^2\frac{x}{2}$ 6

14.(a) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then show that $\vec{\nabla}\cdot\left\{\frac{f(r)}{r}\vec{r}\right\}=\frac{1}{r^2}\frac{d}{dr}\{r^2f(r)\}$. 6

(b) Prove that $\vec{\nabla}\cdot(\vec{F}\times\vec{G})=\vec{G}\cdot(\vec{\nabla}\times\vec{F})-\vec{F}\cdot(\vec{\nabla}\times\vec{G})$, where $\vec{F}$ and $\vec{G}$ are vector point functions. 6

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15.(a) Solve by method of undetermined coefficients. 6

$$(D^2 - 3D)y = x + e^x \sin x\ , \quad D \equiv \frac{d}{dx}$$

(b) Solve: $\frac{dx}{dt} + \frac{2}{t}(x - y) = 1$; $\frac{dy}{dt} + \frac{1}{t}(x + 5y) = t$ 6

16.(a) If $\vec{F} = \phi \operatorname{grad}\phi$, then show that $\vec{F} \cdot \operatorname{curl} \vec{F} = 0$. 6

(b) Solve: $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10x + \frac{10}{x}$ 6

MATHGE-IV

GROUP THEORY

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Define normal subgroup of a group. 3
2. Find all cyclic subgroups of the group $(\mathbb{Z}_7, +)$. 3
3. Show that identity and inverse of an element in a group *G* are unique. 3
4. Show that (6 5 4 3 1 2) is an odd permutation. Find the images of 3 and 4 if $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & & & 3 \end{pmatrix}$ be an even permutation. 1+2
5. Show that any infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$. 3
6. Show that centre of a group *G* is a subgroup of *G*. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Define subgroup of a group *G*. Let *H*, *K* be two subgroups of a group *G*. Prove that *HK* is a subgroup of *G* if and only if *HK* = *KH*. 1+5
8. Show that every cyclic group is commutative. Is the converse true? Justify your answer. 4+2
9. Let *H* be a subgroup of *G* and let $a \in G$. Then show that $aH = H$ if and only if $a \in H$. 6
10. State and prove Lagrange's theorem. 6
11. Find all the homomorphisms from $\mathbb{Z}_{20} \longrightarrow \mathbb{Z}_8$. How many of them are onto? 5+1
12. Find all the subgroups of $\mathbb{Z}/12\mathbb{Z}$. Find the subgroup lattice of D_4. 3+3

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GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Show that $G=\{1, -1, i, -i\}$ forms an abelian group with respect to multiplication. 6

(b) In a group $(G, *)$, if $b^5 = e_G$ and $b*a*b^{-1}=a^2$, $\forall\, a, b \in G$, find the order of 'a'. 6

14.(a) Let $\phi:(G, \circ) \to (G', *)$ be a homomorphism. Prove that $\ker\phi$ is a normal subgroup of G. 4

(b) Prove that order of $U(n)$ is even for $n \geq 3$. 2

(c) Find all elements of order 5 in ($\mathbb{Z}_{40}$, +). Find all the cyclic subgroups of ($\mathbb{Z}_9$, +). 6

15.(a) If H be a subgroup of a cyclic group G, then prove that the quotient group G/H is cyclic. 6

(b) Prove that every permutation on a finite set is either a cycle or a product of disjoint cycles. 6

16. Let H be a subgroup of G. Then show that the following conditions are equivalent:

(a) H is a normal subgroup of G. 4

(b) $gHg^{-1} \subseteq H$, $\forall\, g \in G$ 4

(c) $gHg^{-1} = H$, $\forall\, g \in G$. 4

MATHGE-V

NUMERICAL METHODS

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Show that $\Delta \log f(x) = \log\left[1+\dfrac{\Delta f(x)}{f(x)}\right]$.

2. Find the number of significant figures in:

   (i) $V_A = 11.2461$ given its absolute error as 0.25×10^{-2}.

   (ii) $V_T = 1.5923$ given its relative error as 0.1×10^{-3}.

3. State the condition for convergence of Gauss-Seidel method for solving a system of equations. Are they necessary and sufficient?

4. Given the set of values of $y = f(x)$:

x	2	4	6	8	10
y	5	10	17	29	50

Form the diagonal difference table and find $\Delta^2 f(6)$.

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5. What is the geometric representation of the Newton-Raphson method?

6. Find the function whose first difference is e^x taking the step size $h=1$.

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. The equation $x^2+ax+b=0$ has two real roots α, β. Show that the iteration method $x_{k+1}=-\frac{ax_k+b}{x_k}$ is convergent near $x=\alpha$, if $|\alpha|>|\beta|$. 6

8. What is interpolation? Establish Lagrange's polynomial interpolation formula. 1+5

9. Using the method of Newton-Raphson, find the root of $x^3-8x-4=0$ which lies between 3 and 4, correct upto 4 decimal places. 6

10. Complete the following table: 6

x	10	15	20	25	30	35
$f(x)$	19.97	21.51	—	23.52	24.65	—

11. Use Euler's method, solve the following problem for $x=0.1$ by taking $h=0.02$. 6

$$\frac{dy}{dx}=\frac{y-x}{y+x} \quad \text{with} \quad y(0)=1$$

12. Calculate the approximate value of $\int_0^{\pi/2} \sin x\, dx$, by Trapezoidal rule using 11 ordinates. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Given the following table: 3+3

x	0	5	10	15	20
$f(x)$	1.0	1.6	3.8	8.2	15.4

Construct the difference table and compute $f(21)$ by Newton's Backward formula.

(b) Solve the system by Gauss-Jacobi iteration method: 6

$$x+y+4z=9$$
$$8x-3y+2z=20$$
$$4x+11y-z=33$$

14.(a) Evaluate $\int_0^{\pi/2} \sqrt{1-0.162\sin^2\phi}\, d\phi$, by Simpson's $\frac{1}{3}$rd rule, correct upto four decimal places, taking six sub-interval. 6

(b) Show that Bisection method converges linearly. 6

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15.(a) Use Picard's method to compute $y(0.1)$ from the differential equation $\frac{dy}{dx}=1+xy$ given $y=1$, where $x=0$. 6

(b) Define the operator Δ. Prove that $\Delta^n\left(\frac{1}{x}\right)=\frac{(-1)^n\, n!}{x(x+1)(x+2).....(x+m)}$. 1+5

16.(a) Establish Newton's forward interpolation formula. 6

(b) Use Gauss-elimination method to solve the following: 6

$$-10x_1+6x_2+3x_3+100=0$$
$$6x_1-5x_2+5x_3+100=0$$
$$3x_1+6x_2-10x_3+100=0$$

Correct upto three significant figures.

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