UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2022

CC5-MATHEMATICS

THEORY OF REAL FUNCTIONS AND INTRODUCTION TO METRIC SPACE

Time Allotted: 2 Hours

Full Marks: 60

The figures in the margin indicate full marks

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4=12$

(a) A function $f:[0,1]\to[0,1]$ is continuous on $[0,1]$. Prove that there exists a point c in $[0,1]$ such that $f(c)=c$. 3

(b) Show that $\lim_{x\to0}\frac{x-|x|}{2}$ does not exist. 3

(c) Prove that if $f(x)$ is continuous at $x=a$ and for every $\delta>0$ there is a point in $|x-a|<\delta$, where $f(x)=0$ then $f(a)=0$. 3

(d) Verify Rolle's theorem for $f(x)=2x^3+x^2-4x-2$. 3

(e) Prove that for two subsets A, B of a metric space (X,d) if $A\subseteq B$, then $\delta(A)\le\delta(B)$. 3

(f) In a metric space (X,d) if $a,b\in X$ and $a\neq b$, then show that there exists open balls S_a and S_b containing a and b respectively such that $S_a\cap S_b=\varphi$. 3

GROUP-B

2. Answer any ***four*** questions from the following: $6\times4=24$

(a) If $g(x)=f(x)+f(1-x)$ and $f''(x)<0$ on $[0,1]$, show that $g(x)$ is monotonically increasing on $\left[0,\frac{1}{2}\right]$ and monotonically decreasing on $\left[\frac{1}{2},1\right]$. 6

(b) Show that $\frac{v-u}{1+v^2}<\tan^{-1}v-\tan^{-1}u<\frac{v-u}{1+u^2}$ if $0<u<v$ and deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$ 4+2

(c) Let $\lim_{x\to a}\varphi(x)=l$ and f is continuous at l. Prove that 6

$$\lim_{x\to a}f(\varphi(x))=f(\lim_{x\to a}\varphi(x))=f(l).$$

(d) State and prove Darboux's theorem. 6

(e) Let (X, d) be a complete metric space and Y be a subspace of X. Prove that Y is complete if and only if Y is closed. 6

(f) Let $X = \ell_p \ (1 \le p < \infty) =$ The set of all pth summable sequences of real or complex numbers and let $d(x, y) = \left\{ \sum_{i=1}^{\infty} |x_i - y_i|^p \right\}^{1/p}$ where $x = \{x_n\}$ and $y = \{y_n\} \in \ell_p$. Prove that 'd' is a metric on $X = \ell_p$. 6

GROUP-C

3. Answer any *two* questions from the following: $12 \times 2 = 24$

(a) (i) Find the power series expansion of $\log(1+x)$. Stating clearly its region of validity. 6

(ii) Let (Y, d') be a subspace of a metric space (X, d). Prove that a set $A \subset Y$ is open in (Y, d') if and only if there exists an open set G in (X, d) such that $A = G \cap Y$. 6

(b) (i) Define separable metric space. Give an example of separable metric space with justification. 1+5

(ii) State and prove Cantor's intersection theorem. 1+5

(c) (i) Show that the function f where $f(x)\begin{cases} x\,[1 + \frac{1}{3}\sin(\log x^2)], & x \neq 0 \\ 0, & x = 0 \end{cases}$ 6

is continuous everywhere and monotonic but has no differential coefficient at $x = 0$.

(ii) Let (X, d) be a metric space and $A \subset X$. Show that $\alpha \in \overline{A}$ if and only if $S \cap A \neq \phi$ for every neighbourhood S of α. 6

(d) (i) Use mean value theorem of appropriate order to prove that 6

$x > \sin x > x - \frac{1}{6}x^3, \ 0 < x < \pi/2$.

(ii) Let (X, d) be a metric space. Prove that a non-empty set $A \subset X$ is nowhere dense in X if and only if the set $(\overline{A})^c = X \setminus \overline{A}$ is dense in (X, d). 6

——×——

सामानो मन्त्रः समितिः समानी

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2022

CC6-MATHEMATICS

GROUP THEORY-I

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks

GROUP-A

1. Answer any ***four*** questions: $3\times 4 = 12$

(a) Find the number of elements of order 5 in $\mathbb{Z}_{15}\times \mathbb{Z}_5$.

(b) Let $\beta = (1\ 2\ 3)(1\ 4\ 5)$, write β^{99} in cycle notation.

(c) Prove that a group G in which $a^2 = e$ for every element a in G is a commutative group, where e is an identity element of G.

(d) Find all homomorphism from the group $(\mathbb{Z}_6, +)$ to $(\mathbb{Z}_4, +)$.

(e) Prove that centre of the symmetric group S_3 is trivial.

(f) Prove that a non-abelian group of order 8 must have an element of order 4.

GROUP-B

2. Answer any ***four*** questions: $6\times 4 = 24$

(a) Prove that the order of every subgroup of a finite group G is a divisor of the order of G. 4+2

Is the converse true?

(b) (i) Prove that up to isomorphism, there are only two groups of order 4. 4

(ii) Let $G \neq \{e\}$ be a group of order p^n, p is prime. Show that G contains an element of order p. 2

(c) Let H be a subgroup of a group G and $[G:H] = 2$. Prove that for every $x \in G$, $x^2 \in H$. 4+2

Deduce that A_4 has no subgroup of order 6.

(d) (i) Show that a group G of even order contains an odd number of elements of order 2. 4

(ii) Let in a group G, a be an element of order 30. Find $o(a^{18})$. 2

(e) In the direct product $\mathbb{Z}_{20}\times \mathbb{Z}_{2}\times \mathbb{Z}_{8}\times \mathbb{Z}_{6}$, how many elements of order 12 exists? 6

(f) (i) Let G be a finite commutative group of order n and $\gcd(m, n)=1$. Prove that $\phi: G\to G$ defined by $\phi(x)=x^m$, $x\in G$ is an isomorphism. 3

(ii) Prove that the external direct product of two groups A and B is commutative if and only if both groups A and B are commutative. 3

GROUP-C

$12\times 2=24$

3. Answer any *two* questions:

(a) (i) Let H be a subgroup of a group G. Then prove that $K=\bigcap_{g\in G} gHg^{-1}$ is a normal subgroup of G. 4

(ii) Let $\alpha=\begin{pmatrix}1&2&3&4&5&6&7\\6&4&7&5&2&3&1\end{pmatrix}$, $\beta=\begin{pmatrix}1&2&3&4&5&6&7\\1&4&6&7&3&5&2\end{pmatrix}$ be the elements of S_7. 3+3+2

(A) Write α as a product of disjoint cycles.

(B) Write β as a product of 2-cycles.

(C) Is α^{-1} an even permutation?

(b) (i) Let G and G' be two groups and $\phi: G\to G'$ be onto homomorphism. If $H=\ker\phi$, then prove that $G/H\simeq G'$. 6

(ii) Show that a finite semigroup in which cancellation laws hold is a group. 6

(c) (i) If N and M are normal subgroups of G, then prove that 3+3

(A) $N\cap M$ is also normal in G.

(B) NM is also normal in G.

(ii) Prove that $\mathbb{Z}/3\mathbb{Z}\simeq\mathbb{Z}_3$. 3

(iii) Find the order of H if H is a proper subgroup of a group of order 68 and H is non-cyclic. 3

(d) (i) Prove that the group $S_n (n\geq 3)$ is not abelian. 6

(ii) Show that a group homomorphism $\psi:(G,\circ)\to(H,*)$ is one-one if and only if $\ker\psi=\{e\}$. Deduce that the homomorphism $\phi:(\mathbb{Z}_6,+)\to(\mathbb{Z}_6,+)$ defined by $\phi(\overline{x})=\overline{2x}$ is not one-one. 4+2

——×——

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2022

CC7-MATHEMATICS

RIEMANN INTEGRATION AND SERIES OF FUNCTIONS

Time Allotted: 2 Hours

Full Marks: 60

The figures in the margin indicate full marks

GROUP-A

1. Answer any ***four*** questions: $3\times 4 = 12$

(a) Consider the function: $f(x)=\begin{cases}\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$. 3

Show that the improper integral $\int_{-1}^{1} f(x)dx$ does not exist whereas its Cauchy principle value exists.

(b) Show that $\Gamma(x) > \frac{1}{e}\int_0^1 t^{x-1}dt = \frac{1}{e^x}$ for $x>0$. 3

(c) Examine the uniform convergence of the series $\sum \frac{(-1)^n}{n}|x|^n$ in $-1\leq x\leq 1$. 3

(d) Find the radius of convergence and exact interval of convergence of the power series $\sum \frac{(n+1)}{(n+2)(n+3)}x^n$. 3

(e) Express $f(x)=\frac{\pi - x}{2}$ in a Fourier series in $0<x<2\pi$. 3

(f) Show that the function 3

$$f(x)=\begin{cases}0, & \text{when } x \text{ is rational}\\ 1, & \text{when } x \text{ is irrational}\end{cases}$$

is not integrable on any interval.

GROUP-B

Answer any *four* questions $6\times 4 = 24$

2. Test the convergence of $\int_0^{\infty} \frac{x\,dx}{1+x^4\cos^2 x}$. 6

3. Find the Fourier series for the function $f(x)=\left|\cos\left(\frac{\pi x}{l}\right)\right|$ of period $2l>0$. 6

4. (a) Consider the function f defined on $\left[0, \frac{\pi}{2}\right]$ as follows: 3

$$f(x)=\begin{cases}\cos^2 x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational}\end{cases}$$

Examine the Riemann integrability of f on $\left[0, \frac{\pi}{2}\right]$.

(b) Give an example of a function which is integrable but has no anti-derivative. 3

5. Show that the sequence of functions $\{f_n\}$, where $f_n(x)=x^n$, is uniformly convergent in $[0, k]$, $k<1$ and pointwise convergent in $[0, 1]$. 6

6. Discuss the convergence of $\int_0^{\pi/2} \log(\sin x)dx$. Hence find its value, if possible. 3+3

7. Let $\sum_{n=0}^{\infty} a_n x^k$ be a power series with radius of convergence $R(>0)$ and $f(x)$ be the sum of the series on $(-R, R)$, show that $f^k(0)=k!a_k$ $(k=0, 1, 2, \cdots)$. 6

GROUP-C

Answer any *two* questions 12×2 = 24

8. (a) Show that $\int_0^{\pi/2} \frac{x^m}{\sin^n x}dx$ is convergent iff $n<1+m$. 4

(b) A sequence of functions $\{f_n\}$ is defined by $f_1(x)=\sqrt{x}$, $f_{n+1}(x)=\sqrt{x f_n(x)}$ $\forall n \geq 1$. 4

Show that $\{f_n\}$ is uniformly convergent on $[0, 1]$.

(c) Prove that two different power series cannot converge on the same interval $(-R, R)$, $R>0$ to the same function f. 4

9. (a) Obtain the Fourier series expansion of $f(x)=x\sin x$ on $[-\pi, \pi]$. Hence deduce that $\frac{\pi}{4}=\frac{1}{2}+\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\cdots$. 7

(b) Assuming $\sin^{-1}x = x+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1.3}{2.4}\cdot\frac{x^5}{5}+\cdots$ for $-1 \leq x \leq 1$, prove that 5

$$\int_0^1 \frac{\sin^{-1}x}{x}dx = 1+\frac{1}{2}\cdot\frac{1}{3^2}+\frac{1.3}{2.4}\cdot\frac{1}{5^2}+\cdots$$

10.(a) State and prove Darboux's Theorem. 1+5

(b) If f is bounded and integrable in $[-\pi, \pi]$ and monotonic in $[-\delta, 0)$ and $(0, \delta]$, where $0 < \delta < \pi$, then show that $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n = \frac{f(0^-) + f(0^+)}{\pi} \int_0^{\infty} \frac{\sin x}{x} dx$ where $a_n (n = 0, 1, 2, \cdots)$ denote the Fourier coefficients of f. 6

11.(a) Test the convergence of $\int_0^{\infty} e^{-x} x^{n-1} dx$. 7

(b) Prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n(1+x^n)}$ is uniformly convergent on $[0, 1]$. 5

——×——

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Programme 3rd Semester Examination, 2022

DSC1/2/3-P3-MATHEMATICS

REAL ANALYSIS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

বিভাগ-ক

समूह-क

1. Answer any ***four*** questions: 3×4 = 12

যে-কোনো চারটি প্রশ্নের উত্তর দাওঃ

कुनै चार प्रश्नहरूको उत्तर लेख।

(a) Define supremum and infimum of a set and find sup A and inf A where

$$A=\left\{\frac{1}{m}+\frac{1}{n}: m,\, n\in \mathbb{N}\right\}$$

একটি সেটের 'supremum'এবং 'infimum'-এর সংজ্ঞা দাও এবং 'sup A' এবং 'inf A' নির্ণয় করো যেখানে $A=\left\{\frac{1}{m}+\frac{1}{n}: m,\, n\in \mathbb{N}\right\}$

supremum अनि infimum को परिभाषा लेख अनि $A=\left\{\frac{1}{m}+\frac{1}{n}: m,\, n\in \mathbb{N}\right\}$ को sup A अनि inf A को मान निकाल।

(b) Show that $\forall x,\ y\in \mathbb{R}\ (x<y)$ there exists an irrational number s such that $x<s<y$.

সমস্ত $x,\ y\in \mathbb{R}\,(x<y)$ -এর জন্য দেখাও যে অন্ততপক্ষে একটি অমূলদ সংখ্যা 's' পাওয়া যাবে যাতে $x<s<y$.

सबै $x,\ y\in \mathbb{R}\ (x<y)$ को लागी त्यहाँ irrational संख्या s अवस्थित छ जो $x<s<y$ छ।

(c) Prove that union of finite number of closed sets in $\mathbb{R}$ is a closed set.

প্রমাণ করো যে, বদ্ধ সেট (closed set)-এর সসীম সংখ্যার ইউনিয়ন একটি বদ্ধ সেট (closed set) হবে $\mathbb{R}$-এর মধ্যে।

$\mathbb{R}$ मा closed सेटहरूको सिमीत संख्याहरूको union एउटा closed सेट हो भनी प्रमाण गर।

(d) Show that the series

$$\sum\frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots.$$

does not converges.

দেখাও যে, $\sum\frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots.$ এই শ্রেণী (series)-টি অভিসারী শ্রেণী হবে না।

श्रृंखला $\sum\frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots.$ अभिकेन्द्रित हुदैन भनी प्रमाण गर।

(e) Prove that $\lim\limits_{n\to\infty}(\sqrt{n+1}-\sqrt{n})=0$.

প্রমাণ করো যে, $\lim\limits_{n\to\infty}(\sqrt{n+1}-\sqrt{n})=0$.

$\lim\limits_{n\to\infty}(\sqrt{n+1}-\sqrt{n})=0$ हुन्छ भनी प्रमाण गर।

(f) Determine the nature of the series $\sum\limits_{n=1}^{\infty}(-1)^{n-1}$.

প্রকৃতি নির্ধারণ করো এই $\sum\limits_{n=1}^{\infty}(-1)^{n-1}$ শ্রেণীর (series)

श्रृंखला $\sum\limits_{n=1}^{\infty}(-1)^{n-1}$ को प्रकृति निर्णय गर।

GROUP-B
বিভাগ-খ
समूह-ख

Answer any *four* questions $6\times4=24$

যে-কোনো চারটি প্রশ্নের উত্তর দাও

कुनै चार प्रश्नहरूको उत्तर लेख

2\. (a) Use Cauchy's root test to investigate the nature of the series 4

$$\sum_{n=1}^{\infty}\frac{\left(\frac{n+\sqrt{n}}{2}\right)^n}{n^{n+1}}$$

'Cauchy's root test' ব্যবহার করে, এই $\sum\limits_{n=1}^{\infty}\frac{\left(\frac{n+\sqrt{n}}{2}\right)^n}{n^{n+1}}$ শ্রেণী (series)-এর প্রকৃতি নির্ণয় করো।

श्रृंखला (series) $\sum\limits_{n=1}^{\infty}\frac{\left(\frac{n+\sqrt{n}}{2}\right)^n}{n^{n+1}}$ को प्रकृति Cauchy को root test द्वारा निर्णय गर।

(b) Let $\sum u_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ Find the partial sum S_n of the series $\sum u_n$. 2

ধরা যাক, $\sum u_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ এই শ্রেণী $\sum u_n$ -এর আংশিক যোগফল (Partial Sum) S_n বের করো।

मानौ $\sum u_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ शृंखला $\sum u_n$ को आंशिक योगफल (Partial Sum) निर्णय गर।

3. (a) Prove that every bounded sequence of real number has a convergent subsequence. 4

প্রমাণ করো যে, একটি বাস্তব সংখ্যার প্রতিটি আবদ্ধ অনুক্রমের (bounded sequence) একটি উপ অভিসারী অনুক্রম (convergent subsequence) থাকবে।

प्रत्येक वास्तविक संख्याको सिमाबद्ध अनुक्रम को एउटा अभिकेन्द्रित उप सिमाबद्ध अनुक्रम पाउँछ अनी प्रमाण गर।

(b) Examine the convergence of the sequence 2

$$\frac{2}{3}, \left(\frac{2}{3}\right)^2, \left(\frac{2}{3}\right)^3 \ldots$$

অনুক্রমের অভিসরণ পরীক্ষা করোঃ

$$\frac{2}{3}, \left(\frac{2}{3}\right)^2, \left(\frac{2}{3}\right)^3 \ldots$$

अनुक्रम $\frac{2}{3}, \left(\frac{2}{3}\right)^2, \left(\frac{2}{3}\right)^3 \ldots$ अभिकेन्द्रित हो अनी जाँच गर।

4. Show that if x and y are two numbers of bounded sets of real number S_1 and S_2 respectively, then prove that the set S, whose elements are of the form $x+y$ is also bounded and 6

$$\sup S_1 + \sup S_2 = \sup S$$

যদি x এবং y যথাক্রমে S_1 এবং S_2 বাস্তব আবদ্ধ সেট (real bounded set)-এর দুটি সংখ্যা হয়, তাহলে প্রমাণ করো যে সেই সেট S-এর উপাদানগুলি (elements) $x+y$ আকারের সে সীমাবদ্ধ (bounded) হবে এবং

$$\sup S_1 + \sup S_2 = \sup S .$$

यदि x अनि y क्रमै संगले S_1 अनि S_2 वास्तविक संख्याको सिमावद्ध सेटहरूको दुई संख्याहरू भए, सेट S जसको element हरू $x+y$ रूपमा छ, पनि सिमाबद्ध हुन्छ भनि प्रमाण गर अनि

$$\sup S_1 + \sup S_2 = \sup S$$

5. Prove that $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots$ converges for $p>1$ and diverges for $p\le 1$. 6

প্রমাণ করো, $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots$ একটি অভিসারী শ্রেণী হবে $p>1$-এর জন্য এবং $p\le 1$-এর জন্য অপসারী (divergent) শ্রেণী হবে।

$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots$ $p>1$ को लागी converge गर्छ अनि $p\le 1$ को diverge गर्छ भनी प्रमाण गर।

6. Prove that K be a compact set in $\mathbb{R}$, every infinite subset of K has a limit point in K. Hence, prove that $\mathbb{R}$ is not compact. 4+2

একটি $\mathbb{R}$-এর 'compact' সেট K-এর জন্য, K-এর প্রতিটি অসীম উপসেটের একটি সীমা বিন্দু (limit point) থাকবে K-এর মধ্যে।

সুতরাং, প্রমাণ করো যে '$\mathbb{R}$' compact নয়।

प्रमाण गर $\mathbb{R}$ मा K एउटा compact सेट हो भने प्रत्येक K की उपसेट को एउटा सिमा बिन्दु K मा छ। $\mathbb{R}$ compact होइन भनी पनि प्रमाण गर।

7. (a) Prove that a sequence can have at most one limit. 2

প্রমাণ করো যে, একটি অনুক্রম (sequence)-এর সর্বাধিক একটি 'limit' থাকতে পারে।

एउटा अनुक्रमको कस्तीमा पनि एउटा limit छ भनी प्रमाण गर।

(b) State and prove Heine-Borel theorem. 4

বর্ণনা এবং প্রমাণ করো 'Heine-Borel theorem'.

Heine-Borel उपपाद्य को उल्लेख भनि प्रमाण गर।

GROUP-C

বিভাগ-গ

समूह-ग

Answer any *two* questions 12×2 = 24

যে-কোনো দুটি প্রশ্নের উত্তর দাও

कुनै दुई प्रश्नहरूको उत्तर लेख

8. (a) Show that if a series $\sum x_n$ in $\mathbb{R}$ converges then $x_n\to 0$ as $n\to\infty$. 3

যদি $\mathbb{R}$ একটি শ্রেণী (series) $\sum x_n$ অভিসারী হয়, তবে দেখাও যে $x_n\to 0$ as $n\to\infty$

यदि एउटा श्रृंखला $\sum x_n$ $\mathbb{R}$ मा converge गर्छ भने $n\to\infty$ हुँदा $x_n\to 0$ हुन्छ भनी प्रमाण गर।

(b) Test the convergence: 4

$$\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^2}+\frac{1}{3^5}+\ldots$$

অভিসারীতা (convergence) পরীক্ষা করোঃ

$$\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^2}+\frac{1}{3^5}+\ldots$$

$\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^2}+\frac{1}{3^5}+\ldots$ को अभिकेन्द्रन को जाँच गर।

(c) Prove that a set is closed in $\mathbb{R}$ iff it contains all its limit points. 5

প্রমাণ করো যে, একটি সেট $\mathbb{R}$-এর মধ্যে বদ্ধ হবে যদি এবং শুধুমাত্র যদি এটি সমস্ত limit point-কে নিজের মধ্যে ধারণ করে।

प्रमाण गर एउटा सेट $\mathbb{R}$ मा closed भए यदि अनि यदि मात्र यसले सबै सिमा बिन्दुहरू आफैंमा समावेश गर्छ।

9. (a) Prove that the set $S\subseteq\mathbb{R}$ is closed iff $S'\subset S$. 4

প্রমাণ করো যে সেট $S\subseteq\mathbb{R}$ একটি বদ্ধ (closed) সেট হবে $\mathbb{R}$-এর মধ্যে যদি এবং শুধুমাত্র যদি $S'\subset S$ হয়।

सेट $S\subseteq\mathbb{R}$ closed हुन्छ यदि अनि यदि मात्र $S'\subset S$ हुन्छ भनी प्रमाण गर।

(b) Prove that / প্রমাণ করো / प्रमाण गर 4

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots+\frac{1}{\sqrt{n^2+n}}\right)=1$$

(c) Prove that the sequence $\{(-1)^n\}$ is not a Cauchy sequence. 4

প্রমাণ করো যে, এই অনুক্রম (sequence) $\{(-1)^n\}$ -টি 'Cauchy sequence' হবে না।

अनुक्रम $\{(-1)^n\}$ Cauchy sequence होइन भनी प्रमाण गर।

10.(a) Find all the limit point of the set 4+2

$$S=\left\{\frac{(-1)^m}{m}+\frac{1}{n}\mid m,n\in\mathbb{N}\right\}$$

Is S closed? Is S an open set? Justify your answer.

$S=\left\{\frac{(-1)^m}{m}+\frac{1}{n}\mid m,n\in\mathbb{N}\right\}$ সেটের সমস্ত 'limit point'-গুলো বের করো। S কি বদ্ধ (closed)? S কি একটি open সেট হবে? তোমার মত যাচাই করো।

सेट $S=\left\{\frac{(-1)^m}{m}+\frac{1}{n}\mid m,n\in\mathbb{N}\right\}$ को सबै सिमा बिन्दुहरू खोज। के S closed हो ? के S open set हो ? आफ्नो उत्तरको न्यायोचित गर।

(b) Prove that for any $\varepsilon > 0$, there exist a natural number η such that $\frac{1}{\eta} < \varepsilon$. 4

দেখাও যে, যে কোনো $\varepsilon > 0$ -এর জন্য অন্ততপক্ষে একটি বাস্তব সংখ্যা η পাওয়া যাবে যাতে $\frac{1}{\eta} < \varepsilon$।

कुनै $\varepsilon > 0$ को लागी त्यहाँ एउटा natural संख्या η अवस्थित छ, जस्तै कि $\frac{1}{\eta} < \varepsilon$ हुन्छ भनी प्रमाण गर।

(c) Show that the set of natural number $\mathbb{N}$ is unbounded above. 2

দেখাও যে, $\mathbb{N}$ উপরে সীমাহীন unbounded হবে।

natural संख्या $\mathbb{N}$ को सेट माथि असीमित छ भनी प्रमाण गर।

11.(a) Show that every point in $I = [3, 7]$ is a cluster point of the set $S = I \cap Q$. 4

দেখাও যে $I = [3, 7]$-এর প্রতিটি বিন্দু সেট $S = I \cap Q$-এর একটি 'cluster point' হবে।

$I = [3, 7]$ मा प्रत्येक बिन्दु सेट $S = I \cap Q$ को cluster बिन्दु हो भनी प्रमाण गर।

(b) Show that the sequence $\{S_n\}$; where $S_n = \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$, $\forall n \in \mathbb{N}$ is convergent. 4

দেখাও যে, অনুক্রম $\{S_n\}$ যেখানে $S_n = \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$, $\forall n \in \mathbb{N}$ টি অভিসারী হবে।

अनुक्रम $\{S_n\}$ जहाँ $S_n = \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$ $\forall n \in \mathbb{N}$ convergent हो भनी प्रमाण गर।

(c) Prove that derived set of bounded set is bounded. 4

দেখাও যে বদ্ধসেট (bounded set)-এর 'derived set' টি বদ্ধ হবে।

सिमाबद्ध सेटको derived सेट, सिमाबद्ध हो भनी प्रमाण गर।

——×——

'আলো পথ সকলি সবাই'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2022

SEC1-P1-MATHEMATICS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks

The question paper contains SEC1A and SEC1B. Candidates are required to answer any *one* from the *two* SEC1 courses and they should mention it clearly on the Answer Book.

SEC1A

LOGIC AND SETS

GROUP-A

1. Answer any *four* questions: $3\times 4 = 12$

(a) If a is an odd integer, establish that $a^2+(a+2)^2+(a+4)^2+1$ is divisible by 12.

(b) Describe the method of contradiction to prove an argument. 3

(c) Show that $(p\wedge q)\rightarrow(p\vee q)$ is a tautology. 3

(d) Determine all solutions in integer of $24x+138y=18$. 3

(e) If $p\geq 5$ is a prime number, show that p^2+2 is composite. 3

(f) Let $A_i=\{1, 2, 3, \cdots, i\}$ for $i=1, 2, \cdots$ then find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$. 3

GROUP-B

2. Answer any *four* questions: $6\times 4 = 24$

(a) State Fermat's theorem. Using induction prove that if p is a prime, then $a^p \equiv a(\text{mod } p)$ for any integer a. 6

(b) For three sets A, B, C show that $A\cap(B\Delta C)=(A\cap B)\Delta(A\cap C)$. 6

(c) Find the number of non-negative integer solutions of the inequality $x_1+x_2+x_3+x_4+x_5+x_6<10$ where $x_i\geq 0$, $i=1, 2, \ldots, 6$ 6

(d) Find the H.C.F and L.C.M of the numbers 12, 20 and 140 using set theory. 6

(e) Show that each integer divisor $e>1$ of a^2+b^2 is a product of Gaussian prime divisors $q+ir$ of a^2+b^2, unique up to unit factors. 6

(f) (i) Find the negation of the following statement: 3

$\forall x \, \exists y[\{p(x,y)\wedge q(x,y)\}\Rightarrow r(x,y)]$

(ii) Establish the validity of the argument: 3

$$\begin{array}{l} p\vee q \\ p\Rightarrow \sim q \\ p\Rightarrow r \\ \hline \therefore r \end{array}$$

GROUP-C

Answer any *two* questions $12\times 2=24$

3. (a) How many relations are there on a set with n elements? How many of them are reflexive relations? 6

(b) Define POSET. Show that the relation "$\geq$" is a partial ordering on $\mathbb{Z}$. 6

4. (a) If A and B be two equivalence relations on a set S then prove that $A\bigcap B$ is an equivalence relation. 6

(b) Prove that if n is an integer then $n^2\geq n$. 6

5. (a) Explain tautology and contingency. Construct truth tables to determine whether the following statements are tautology or contingency: 6

(i) $\{p\Rightarrow(q\wedge r)\}\Rightarrow\sim(p\Rightarrow q)$

(ii) $\sim(p\wedge\sim q)\Leftrightarrow\sim p\vee q$.

(b) Prove that $A-\left(\bigcup_{i=1}^{n}B_i\right)=\bigcap_{i=1}^{n}(A-B_i)$. Show how this formula is the generalization of the De Morgan's law. 6

6. (a) State and prove Euler's criterion. 8

(b) Verify that if p is an odd prime, then 4

$$\left(\frac{-2}{p}\right)=\begin{cases}1, & \text{if } p\equiv 1 \pmod 8 \text{ or } p\equiv 3 \pmod 8\\ -1, & \text{if } p\equiv 5 \pmod 8 \text{ or } p\equiv 7 \pmod 8\end{cases}$$

SEC1B

C++

GROUP-A

1. Answer any *four* questions: $3\times4=12$

(a) What will be the output of the following program?

```
# include <iostream>
   void main( )
      {
         int a, b, c = 50;
         float age;
         a = c;
         b = c + 50;
         age = 23;
         cout << "a = " <<a << "\n";
         cout << "b = " <<b << "\n";
         cout << "c = " <<c << "\n";
         cout << "age = " << age;
      }
```

(b) What is an inline function? Is it possible to ignore inlining?

(c) Write a C++ program to find the maximum of two input numbers.

(d) What are the most important differences between C and C++ ?

(e) Write a C++ program to find the absolute value of an integer.

(f) What is friend function? Describe its importance.

GROUP-B

Answer any *four* questions $6\times4=24$

2. Suppose there are two '.txt' files named file1.txt and file2.txt. Write a C++ program that reads the data file1.txt and copy every alternative character to file2.txt.

3. Write a C++ program to alternate rows and columns of a 4×4 matrix.

4. Which header file requires to calculate the length of a string? Write a C++ program to calculate the length of a string.

5. Write a C++ program that will give the following output:

```
1 2 3 4 5
1 2 3 4
1 2 3
1 2
1
```

6. What is inheritance? What are base and derived classes? Give a suitable example for inheritance. 2+2+2

7. What is demonstrated by the following program? 6

```
# include < iostream >
    using namespace std;
    int main( )
    {   int exponent;
        float base, result = 1;
        cout ≪"Enter base and exponent:";
        cin ≫ base ≫ exponent;
        cout ≪ base ≪ "∧"≪exponent ≤ "=";
        while (exponent != 0 )
            {
            result *= base;
            – – exponent;
            }
        cout << result;
        return 0;
    }
```

GROUP-C

Answer any *two* questions $12\times2 = 24$

8. (a) Write a C++ program to detect and handle divide by zero errors. 7

(b) Write a C++ program to find the sum of first 15 even numbers and their squares sum. 5

9. (a) Write a C++ program to find the factorial of a positive integer. 4

(b) Write a C++ program for sorting names in alphabetical order. 8

10.(a) What are copy constructor? Explain their need. 4

(b) Write a program in C++ to illustrates the concept of overriding default operations performed by a user defined copy constructor. 8

11.(a) What is class? Describe the syntax for declaring a class with example. 5

(b) Write a program in C++ to declare a class employee, consisting of data members "employee no" and "employee name". Write the member functions "accept( )" to accept and "display( )" to display the data for five employees. 7

——x——