सत्यानी मन्त्र समिति सत्यानी

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2022

CC1-MATHEMATICS

CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Find $\int \sin^4 x \cos^2 x \, dx$.

(b) Find the points of inflexion, if any, of the curve $x = (\log y)^3$.

(c) Obtain reduction formula for $\int \tan^n x \, dx$, n being a positive integer, greater than 1.

(d) Obtain the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$; $2x + 3y + 4z = 8$ is a great circle.

(e) Find the solution of the differential equation $(x^2 + y^2 + 2x)\,dx + 2y\,dy = 0$.

(f) Find the differential equation of all circles, which pass through the origin and whose centres are on the x-axis.

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) (i) State Leibnitz's theorem on successive derivatives. 2

(ii) If $y = (\sin^{-1} x)^2$, prove that 4

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$$

(b) Find the volume of the solid generates by revolving the cardioid $r = a(1 - \cos\theta)$, about the initial line. 6

(c) If 'g' is a variable tangent of the conic $\frac{l}{r} = 1 - e\cos\theta$, show that the locus of the foot of the perpendicular from the pole on 'g' is the circle $r^2(e^2 - 1) + 2elr\cos\theta + l^2 = 0$. 6

(d) (i) Show that $\frac{1}{x^2}$ is an integrating factor of $x\,dy - y\,dx = 0$. 3

(ii) Solve the differential equation 3

$$(xy^2 - e^{1/x^3})dx - x^2 y\, dy = 0$$

(e) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx$, show that $2(n-1)I_n = 1 + (n-2)I_{n-1}$ and hence deduce that $I_n = \frac{1}{n-1}$. 6

(f) Solve the differential equation 3+3

(i) $\left(x + y\cos\frac{y}{x}\right)dx = x\cos\frac{y}{x}\,dy$

(ii) $\frac{dy}{dx} = \sin(x+y)$

GROUP-C

3. Answer any *two* questions from the following: 12×2 = 24

(a) (i) Find the rectilinear asymptotes of the curve 6+6

$$x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 = 2.$$

(ii) If $I_n = \int_0^1 x^n \tan^{-1} x \, dx$, then show that

$$(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$$

(b) (i) Find the values of α, β such that 4+4+4

$$\lim_{x\to 0} \frac{\alpha\sin 2x - \beta\sin x}{x^3} = 1$$

(ii) Find $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$

(iii) If $y = \sin ax$, a is constant, then show that

$$\frac{d^n y}{dx^n} = a^n \sin\left(ax + \frac{n\pi}{2}\right), \; n \in \mathbb{N}.$$

(c) (i) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ about its base. 6+6

(ii) Solve the differential equation

$$\frac{dy}{dx} + \frac{1}{(1+x^2)}y = \frac{e^{\tan^{-1}x}}{1+x^2}$$

(d) Solve the differential equation 6+6

(i) $x^3\frac{dy}{dx} = y^3 + y^2\sqrt{(y^2 - x^2)}$

(ii) $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$

———x———

UNIVERSITY OF NORTH BENGAL

B Sc. Honours 1st Semester Examination, 2022

CC2-MATHEMATICS

ALGEBRA

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) If $T: \mathbb{R}^3 \to \mathbb{R}$ and $T(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$ then show that T is not a Linear Transformation. 3

(b) If a, b, x are real and $\text{mod}(a+ib) = 1$, prove that $(a+ib)^{ix}$ is purely real. 3

(c) A relation ρ on $\mathbb{Z}$ is defined by $\rho = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a - b \text{ is divisible by } 7\}$. Show that ρ is an equivalence relation. 3

(d) If $x^3 + 3px + q$ has a factor of the form $(x-\alpha)^2$, then show that $q^2 + 3p^3 = 0$. 3

(e) Prove that $n(n+1)^2 > 4(n!)^{3/n}$ where n be a positive integer greater than 1. 3

(f) Determine the rank of the matrix 3

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}.$$

GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) Obtain the fully reduced normal form of the matrix 6

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

(b) If $\log \sin(x+iy) = u + iv \ (0 < x < \pi)$, prove that 6

(i) $u = \frac{1}{2}\log(\cosh^2 y - \cos^2 x)$

(ii) $v = \tan^{-1}(\cot x \tanh y)$

(c) If α be a non-real root of $x^7 = 1$, find the equation whose roots are 6

$$(\alpha + \alpha^6), (\alpha^2 + \alpha^5), (\alpha^3 + \alpha^4).$$

(d) Find the eigenvalues and the corresponding eigenvectors of the following real matrix. 6

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 1 \\ 3 & 5 & 6 \end{pmatrix}$$

(e) If a, b, c, d are positive and not all equal then prove that 6

$$\frac{3}{b+c+d}+\frac{3}{c+d+a}+\frac{3}{d+a+b}+\frac{3}{a+b+c}>\frac{16}{a+b+c+d}.$$

(f) (i) Find the least positive residues in $3^{16} \pmod{77}$. 3+3

(ii) If a mapping $f: A \to \mathbb{R}$, where $A=\{x \mid 0<x<1\}$ is defined by $f(x)=\frac{2x-1}{1-|2x-1|}$, $x \in A$ then show that f is bijective.

GROUP-C

3. Answer any *two* questions: 12×2 = 24

(a) (i) State and prove Division algorithm. 6

(ii) The matrix of a Linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ relative to the ordered basis $\{(-1, 1, 1), (1, -1, 1), (1, 1, -1)\}$ of $\mathbb{R}^3$ is $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 3 & 3 & 6 \end{pmatrix}$. Find $T(x, y, z)$, where $(x, y, z) \in \mathbb{R}^3$. Is T invertible? 6

(b) (i) Solve the biquadratic equation by Ferrari's method: 6

$$x^4+2x^3-7x^2-8x+12=0.$$

(ii) Prove that the product of any m consecutive integers is divisible by m. 3

(iii) Find all values of $(-i)^{3/4}$. 3

(c) (i) If the roots α, β, γ of the equation $x^3+qx+r=0$ are in A.P., show that the rank of the matrix $\begin{pmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{pmatrix}$ is 2. 4

(ii) If $\alpha_1, \alpha_2, \cdots, \alpha_n$ are n distinct roots of the equation $x^n-1=0$, then prove that $(a+b\alpha_1)(a+b\alpha_2)\cdots\cdots(a+b\alpha_n)=a^n+(-1)^{n-1}b^n$. 5

(iii) Prove that $3.4^{n+1} \equiv 3 \pmod{9}$ for all positive integer n. 3

(d) (i) Determine the conditions for which of the following system of linear equations 6

$$\begin{aligned} x+2y+z &= 1 \\ 2x+y+3z &= b \\ x+ay+3z &= b+1 \end{aligned}$$

has (A) Unique solution, (B) No solution and (C) many solutions.

(ii) Find the inverse of the given matrix A by using Cayley Hamilton theorem: 6

$$A=\begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}$$

——×——

UNIVERSITY OF NORTH BENGAL

B.Sc. Programme 1st Semester Examination, 2022

DSC1/2/3-P1-MATHEMATICS

CALCULUS AND GEOMETRY

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks

GROUP-A / বিভাগ-ক / समूह-क

1. Answer any ***four*** questions of the following: 3×4 = 12

নিম্নলিখিত যে-কোন *চারটি* প্রশ্নের উত্তর দাও:

कुनै चार प्रश्नहरूको उत्तर लेख –

(a) Evaluate $\lim_{x\to 0}\frac{1}{x}(\sqrt{1+x}-\sqrt{1-x})$

মান বের কর: $\lim_{x\to 0}\frac{1}{x}(\sqrt{1+x}-\sqrt{1-x})$

$\lim_{x\to 0}\frac{1}{x}(\sqrt{1+x}-\sqrt{1-x})$ को मान निर्णय गर।

(b) If $u=\sin\alpha x+\cos\alpha x$. Show that $u_n=\alpha^n\{1+(-1)^n\sin 2\alpha x\}^{1/2}$.

যদি $u=\sin\alpha x+\cos\alpha x$ হয়, তাহলে দেখাও যে, $u_n=\alpha^n\{1+(-1)^n\sin 2\alpha x\}^{1/2}$।

यदि $u=\sin\alpha x+\cos\alpha x$ भए, देखाऊ $u_n=\alpha^n\{1+(-1)^n\sin 2\alpha x\}^{1/2}$ ।

(c) Find the area of the curve, $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$.

$\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$ এই বক্ররেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

वक्र $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$ को क्षेत्रफल निकाल।

(d) If $I_n=\int_0^{\pi/2}\sin^n x\,dx$, where 'n' is a positive integer, prove that $I_n=\frac{n-1}{n}I_{n-2}$, for $n>2$.

যদি $I_n=\int_0^{\pi/2}\sin^n x\,dx$ হয় যেখানে 'n' হল একটি ধনাত্মক পূর্ণসংখ্যা, তাহলে প্রমাণ কর $I_n=\frac{n-1}{n}I_{n-2}$ যেখানে $n>2$।

एउटा धनात्मक पूर्णांक 'n' को लागी यदि $I_n=\int_0^{\pi/2}\sin^n x\,dx$ भए, $n>2$ को लागी $I_n=\frac{n-1}{n}I_{n-2}$ हुन्छ भनी देखाऊ।

(e) Find the angle of rotation of the axes for which the equation $x^2 - y^2 = a^2$ will reduce to $xy = c^2$. Determine c^2.

অক্ষের ঘূর্ণন কোণের মান বের কর যার জন্য $x^2 - y^2 = a^2$ সমীকরণটি $xy = c^2$ সমীকরণে রূপান্তরিত হবে। c^2-এর মান বের কর।

समीकरण $x^2 - y^2 = a^2$, $xy = c^2$ मा परिणत हुदा, अक्षहरू घुमेको कोण निर्णय गर। c^2 को मान पनि निर्णय गर।

(f) If $y = e^{m\sin^{-1}x}$, show that $(1-x^2)y_2 - xy_1 - m^2y = 0$.

যদি $y = e^{m\sin^{-1}x}$ হয়, তাহলে দেখাও যে $(1-x^2)y_2 - xy_1 - m^2y = 0$।

यदि $y = e^{m\sin^{-1}x}$ भए, प्रमाण गर $(1-x^2)y_2 - xy_1 - m^2y = 0$.

GROUP-B / বিভাগ-খ / समूह-ख

$6\times4 = 24$

2. Answer any *four* questions from the following:

নিম্নলিখিত যে-কোন *চারটি* প্রশ্নের উত্তর দাও:

कुनै चार प्रश्नहरूको उत्तर लेख –

(a) Find a, b in order that $\lim_{x\to0}\frac{a\sin 2x - b\sin x}{x^3} = 1$.

যদি $\lim_{x\to0}\frac{a\sin 2x - b\sin x}{x^3} = 1$ হয়, তাহলে a এবং b-এর মান নির্ণয় কর।

$\lim_{x\to0}\frac{a\sin 2x - b\sin x}{x^3} = 1$ भए, a, b को मान निर्णय गर।

(b) If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$.

যদি $y^{1/m} + y^{-1/m} = 2x$ হয়, তাহলে প্রমাণ কর যে

$$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$$

यदि $y^{1/m} + y^{-1/m} = 2x$ भए, $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$ हुन्छ भनी प्रमाण गर।

(c) If $I_n = \int_0^1 x^n \tan^{-1}x\,dx, (n>2)$, then prove that $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$.

যদি $I_n = \int_0^1 x^n \tan^{-1}x\,dx, (n>2)$ হয়, তাহলে প্রমাণ কর যে $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$.

यदि $I_n = \int_0^1 x^n \tan^{-1}x\,dx, (n>2)$ भए, प्रमाण गर $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$।

(d) Find the asymptotes of the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$.

$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$ এই বক্ররেখার Asymptotes বের কর।

वक्र $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$ को एसिम्प्टोट् निर्णय गर।

(e) Show that the spheres $x^2+y^2+z^2-2x-4y-4z=0$ and $x^2+y^2+z^2+10x+2z+10=0$ touch each other externally. Find the point of contact.

দেখাও যে $x^2+y^2+z^2-2x-4y-4z=0$ এবং $x^2+y^2+z^2+10x+2z+10=0$ এই গোলকদ্বয় পরস্পরকে বাইরের দিকে স্পর্শ করেছে। উক্ত গোলকদ্বয়ের স্পর্শ বিন্দুটি নির্ণয় কর।

अण्डाकर वृत्तहरू $x^2+y^2+z^2-2x-4y-4z=0$ अनि $x^2+y^2+z^2+10x+2z+10=0$ एकाकी लाई छुन्छ भनी प्रमाण गर।

(f) Trace the curve : $(x+3)(x^2+y^2)=4$.

$(x+3)(x^2+y^2)=4$ এই বক্ররেখাটি অঙ্কন (Trace) কর।

वक्र $(x+3)(x^2+y^2)=4$ को चित्र बनाऊ।

GROUP-C / বিভাগ-গ / समूह-ग

3. Answer any *two* questions from the following: $12\times2=24$

যে-কোন *দুটি* প্রশ্নের উত্তর দাওঃ

कुनै दुईवटा प्रश्नहरूको उत्तर लेख –

(a) (i) Find a reduction formula for $\int \sin^m x\cos^n x\,dx$, where $m, n\in\mathbb{N}$. Hence find a reduction formula for $\int_0^{\pi/2}\sin^m x\cos^n x\,dx$. 6+2

$\int \sin^m x\cos^n x\,dx$-এর হ্রাস সূত্র (reduction formula) বের কর যেখানে $m, n\in\mathbb{N}$. অতঃপর $\int_0^{\pi/2}\sin^m x\cos^n x\,dx$-এর হ্রাস সূত্র (reduction formula) বের কর।

$\int \sin^m x\cos^n x\,dx$, $m, n\in\mathbb{N}$ को लागी reduction सूत्र निर्णय गर। त्यस पछि $\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ को reduction सूत्र निर्णय गर।

(ii) If $y=2\cos x(\sin x-\cos x)$, then find $\frac{d^{10}y}{dx^{10}}$ at $x=0$. 4

যদি $y=2\cos x(\sin x-\cos x)$ হয়, তাহলে $x=0$ বিন্দুতে $\frac{d^{10}y}{dx^{10}}$-এর মান নির্ণয় কর।

यदि $y=2\cos x(\sin x-\cos x)$ भए, $x=0$ मा $\frac{d^{10}y}{dx^{10}}$ को मान निर्णय गर।

(b) (i) Find the nature of the conic $r=\frac{2}{4+\cos\theta}$ and also find the equation of directiries. 6

$r=\frac{2}{4+\cos\theta}$ এই conic-এর nature নির্ণয় কর এবং নিয়ামকের (directrix) সমীকরণ নির্ণয় কর।

कोनिक $r=\frac{2}{4+\cos\theta}$ को प्रकृति निर्णय गर अनि directiries को मान पनि निर्णय गर।

(ii) Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle. 6

গোলকের সমীকরণ নির্ণয় কর যেখানে $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ এই বৃত্তটি একটি great বৃত্ত।

বৃত্त $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ एउटा वृहत्त वृत्त भए भने अण्डा वृत्तको समीकरण निर्णय गर।

(c) (i) Discuss the nature of the conic $4x^2-4xy+y^2+2x-26y+9=0$ and reduce it to its canonical form. 6

$4x^2-4xy+y^2+2x-26y+9=0$ এই conic-টির nature সম্পর্কে আলোচনা কর এবং এটির canonical form তে reduce করে দেখাও।

कोनिक $4x^2-4xy+y^2+2x-26y+9=0$ को प्रकृति छलफल गर्दै यसलाई canonical रूपमा परिणत गर।

(ii) Find the volume of the solid obtained by revolving the cycloid $x=a(\theta+\sin\theta)$, $y=a(1+\cos\theta)$ about its base. 6

$x=a(\theta+\sin\theta)$, $y=a(1+\cos\theta)$ এই Cycloid-টি উহার পাদদেশের সাপেক্ষে আবর্তের ফলে যে ঘনবস্তুটি পাওয়া যাবে তার আয়তন নির্ণয় কর।

आधारको अड़ानमा एउटा साइक्लोइड $x=a(\theta+\sin\theta)$, $y=a(1+\cos\theta)$ लाई घुमाउँदा पाएको ठोस् को मात्रा (volume) निर्णय गर।

(d) (i) Prove that the curves $\frac{x^2}{a}+\frac{y^2}{b}=1$, $\frac{x^2}{a'}+\frac{y^2}{b'}=1$ cut orthogonally if $a-b=a'-b'$. 6

প্রমাণ কর $\frac{x^2}{a}+\frac{y^2}{b}=1$, $\frac{x^2}{a'}+\frac{y^2}{b'}=1$ বক্ররেখাদ্বয় orthogonally ছেদ করবে যদি $a-b=a'-b'$ হয়।

वक्रहरू $\frac{x^2}{a}+\frac{y^2}{b}=1$ अनि $\frac{x^2}{a'}+\frac{y^2}{b'}=1$ लम्बवत् रूपमा (orthogonally) काटिन्छ यदि $a-b=a'-b'$ प्रमाण गर।

(ii) If $y=x^{2n}(n\in\mathbb{N})$, then show that $y_n=2^n[1\ 3\ 5\cdots(2n-1)]x^n$. 6

যদি $y=x^{2n}(n\in\mathbb{N})$ হয়, তাহলে দেখাও যে $y_n=2^n[1\cdot3\cdot5\cdots(2n-1)]x^n$.

यदि $y=x^{2n}$ $(n\in\mathbb{N})$ भए, प्रमाण गर, $y_n=2^n[1\cdot3\cdot5\cdots(2n-1)]x^n$ ।

——×——